1971

# Automatic amplitude and phase equalization derived from frequency-domain data

David Harold Bliss
*Iowa State University*

72-5177

BLISS, David Harold, 1941-
AUTOMATIC AMPLITUDE AND PHASE EQUALIZATION
DERIVED FROM FREQUENCY-DOMAIN DATA.

Iowa State University, Ph.D., 1971
Engineering, electrical

University Microfilms, A XEROX Company, Ann Arbor, Michigan

# Automatic amplitude and phase equalization derived from frequency-domain data

by

David Harold Bliss

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Electrical Engineering

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

For the Major Department

Signature was redacted for privacy.

For the Graduate College

Iowa State University
Ames, Iowa

1971

# TABLE OF CONTENTS


| | | Page |
|---|---|---|
| I. | INTRODUCTION | 1 |
| II. | COMMUNICATION CHANNEL EQUALIZATION | 5 |
| III. | METHOD OF PROCEDURE | 11 |
| IV. | DISCUSSION OF FINDINGS | 28 |
| V. | SUMMARY AND CONCLUSIONS | 73 |
| VI. | LITERATURE CITED | 77 |
| VII. | ACKNOWLEDGEMENTS | 81 |
| VIII. | APPENDIX A: DERIVATION OF ALGORITHM | 82 |
| IX. | APPENDIX B: EQUALIZED CHANNEL ERROR EXPRESSION | 88 |
| X. | APPENDIX C: PROGRAM FLOW DIAGRAM AND LISTING | 91 |

# LIST OF FIGURES


| Figure | | Page |
|---|---|---|
| 1. | Automatic equalizer in data communication system | 4 |
| 2. | Block diagram of tapped delay line equalizer | 6 |
| 3. | C4 grade specifications | 14 |
| 4. | Error weighting functions | 16 |
| 5. | Linear dB attenuation and parabolic delay format, 3500 Hz bandwidth | 29 |
| 6. | Equalizer phase shift function for two algorithms, case 1 | 32 |
| 7. | Equalized channel error vs. ideal channel delay, cases 9 and 10 | 35 |
| 8. | Case 9 characteristic | 36 |
| 9. | Case 10 characteristic | 37 |
| 10. | Equalized channel error vs. ideal channel delay, case 2 | 40 |
| 11. | Equalized channel error vs. delay per section, cases 2 and 4 | 43 |
| 12. | Case 11 amplitude response | 47 |
| 13. | Case 12 characteristic | 51 |
| 14. | Equalized channel error vs. maximum of parabolic delay, cases 1-4 | 54 |
| 15. | Equalized channel error vs. maximum attenuation, cases 5-7 | 58 |
| 16. | Case 8 characteristics | 63 |
| 17. | Case 13 characteristics | 70 |
| 18. | Program EQLR flow diagram | 92 |

# LIST OF TABLES


| Table | | Page |
|---|---|---|
| 1. | Channel distortion cases | 30 |
| 2. | Computed tap gains for distortion case 10 | 39 |
| 3. | Computed tap gains for distortion case 11 | 48 |
| 4. | Computed tap gains for distortion case 12 | 52 |
| 5. | Equalization results for distortion case 1 | 53 |
| 6. | Equalization results for distortion cases 2-4 | 55 |
| 7. | Equalization results for distortion cases 5-7 | 59 |
| 8. | Effect of weighting on equalization results, cases 3 and 7 | 64 |
| 9. | Effect of measurement data spacing on equalization results, cases 4 and 7 | 66 |
| 10. | Additional equalization results for distortion case 4 | 68 |
| 11. | Equalization results for distortion case 13 | 69 |

# I. INTRODUCTION

Many existing communication channels, for example, the voice-band facilities of the vast telephone networks, do not permit digital data transmission speeds commensurate with the available signal-to-noise ratio and the occupied bandwidth. The underlying reason is that the typical channel, described by Alexander _et al_. [1], cannot be realistically represented by a constant-attenuation, linear-phase vs. frequency characteristic, a condition necessary for utilization of the occupied bandwidth to its fullest in generalized data transmission. Departures from these ideal conditions are called linear distortions. Bennett and Davey [5] thoroughly discuss the optimization of data transmission and the effects of linear distortions. Examples of transmission media are open-wire pairs, multi-pair cables, and multiplexed microwave line-of-sight channels. In these cases the distortions are typically caused by the attenuation properties of the medium, by loading introduced to compensate for nonuniform attenuation, by frequency-multiplex separation filters, and by improperly matched hybrid transformers and repeaters. The effect of linear distortions is to modify the received spectrum and, in turn, spread the data signal time waveform. The result, known as intersymbol interference, gives rise to an increased probability of transmission errors.

Even though not designed and installed for data transmission application, voice channels provide the bulk of data communication facilities [5]. Hence, means of compensating for the characteristics are necessary as the volume of data flow increases. A method of reducing the intersymbol interference is to "equalize" the channel, preferably at the receiving terminal, to achieve a flat-amplitude and linear-phase characteristic in the frequency domain. The device employed is termed an "equalizer". The equalization can also be accomplished by minimizing the intersymbol interference at the sampling instants of the signal waveform in the time domain when the signal format is known. The flat-amplitude, linear-phase characteristic channel is suitable for transmission usage by a wide variety of modulation techniques, rather than being constrained to a particular synchronous data format.

Equalization achieved before the data transmission begins is termed "preset" or sometimes "pre-call". Preset equalization should be sufficient in most cases since the typical channel characteristic changes little during an average call [27]. An "adaptive" equalizer responds to changes in the channel characteristic during the data transmission. A general discussion of equalization techniques is given in Chapter II.

The purpose of the research reported here is to develop and evaluate an algorithm for computing the optimumum parameter values of a certain type of preset equalizer given frequency-domain measurements of the channel characteristic. The equalizer is to be utilized in a data communication system as shown in Figure 1. The frequency-domain measurements are assumed to be made in an interval of 1 to 3 minutes after the channel routing has been established. An accurate computer-controlled measurement device capable of accomplishing this, assuming suitable switching automation, has been recently described by Reynolds [38]. This investigation considers the feasibility of converting such measurements into equalizer adjustment values.

The method chosen for evaluating the conversion algorithm is to simulate the equalizer and its control routine using a digital computer program. Special test cases to verify proper solution via the routine are reported, followed by equalization simulation results for typical channel characteristics. The effects of some implementation hardware parameters are investigated. Unique operational features of the scheme are also modeled, to establish the utility of the technique. The tapped delay line equalizer simulation model is identified as program EQLR.

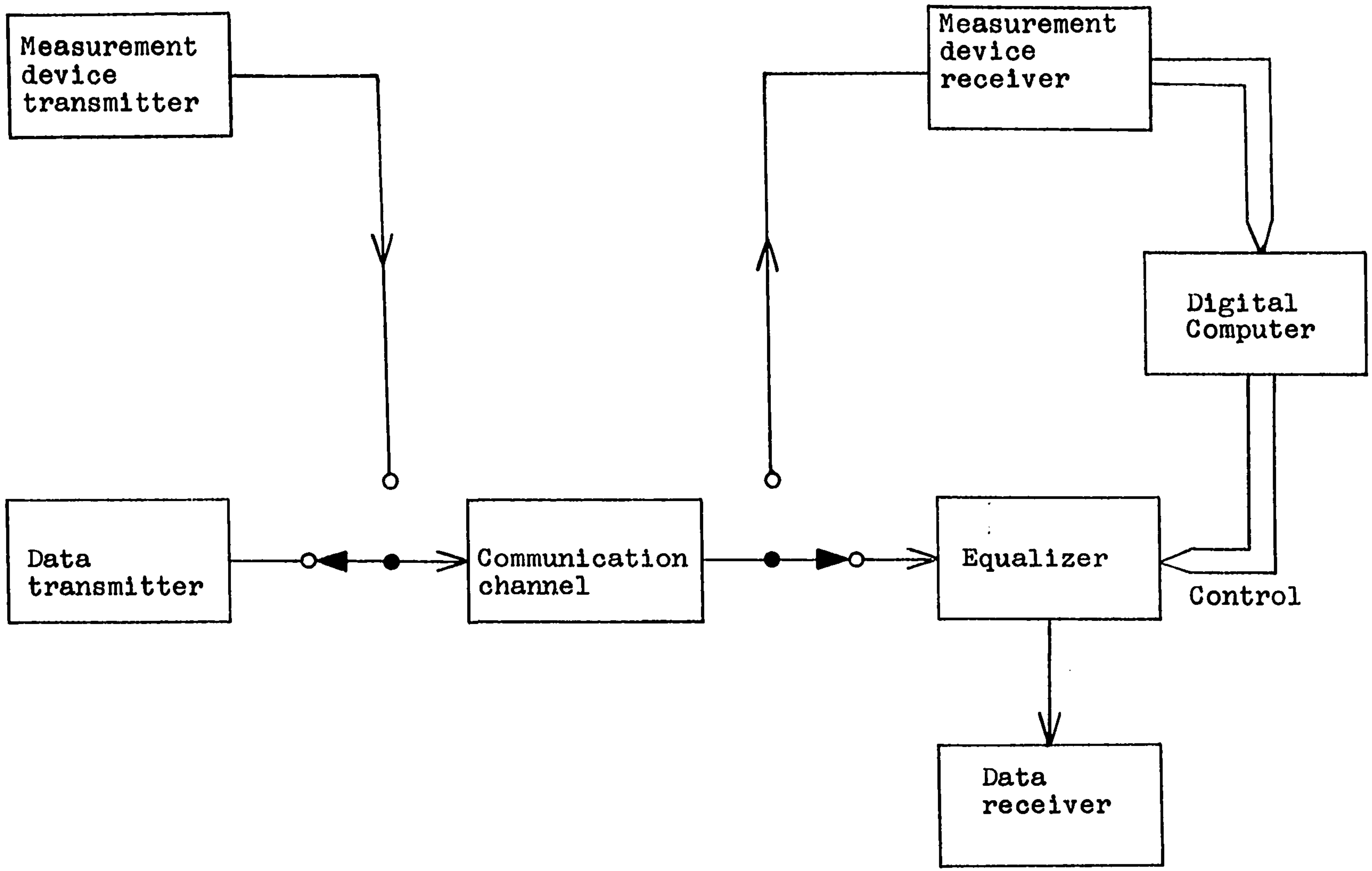


Figure 1. Automatic equalizer in data communication system

## II. COMMUNICATION CHANNEL EQUALIZATION

Manual equalization using a variety of fixed amplitude and delay compensating circuits has been employed for many years [4], [32]. Lang [24] presents an analysis of the use of lattice networks for delay equalization only, using an iterative computer solution of a system of nonlinear equations to determine circuit component values.

Automated equalization is usually accomplished using a tapped delay line, also known as a transversal filter. The block diagram of such a device is shown in Figure 2. Lucky _et al_. [31] discuss the fundamentals of transversal filter theory and applications. The delay line shown has a total of $N=2L+1$ tap points, located at T seconds delay intervals. Each tap point signal is routed through an adjustable gain/attenuation scaler having transmittance of $c_i$, hereafter referred to as the tap gain. It may be either positive or negative. The several scaled signals are collectively passed through a summing amplifier to the output.

The transversal filter was patented by Lee and Wiener [25], [46] in 1936 and later by Blumlein _et al_. [6]. It was first described in the literature by Kallmann [22], in 1940. Boothroyd and Creamer [7] utilized the device for forcing to zero the interference at sampling instants, primarily by experimentally adjusting the tap gains. Bellows and Graham [3] in 1957 applied the transversal filter to equalization in

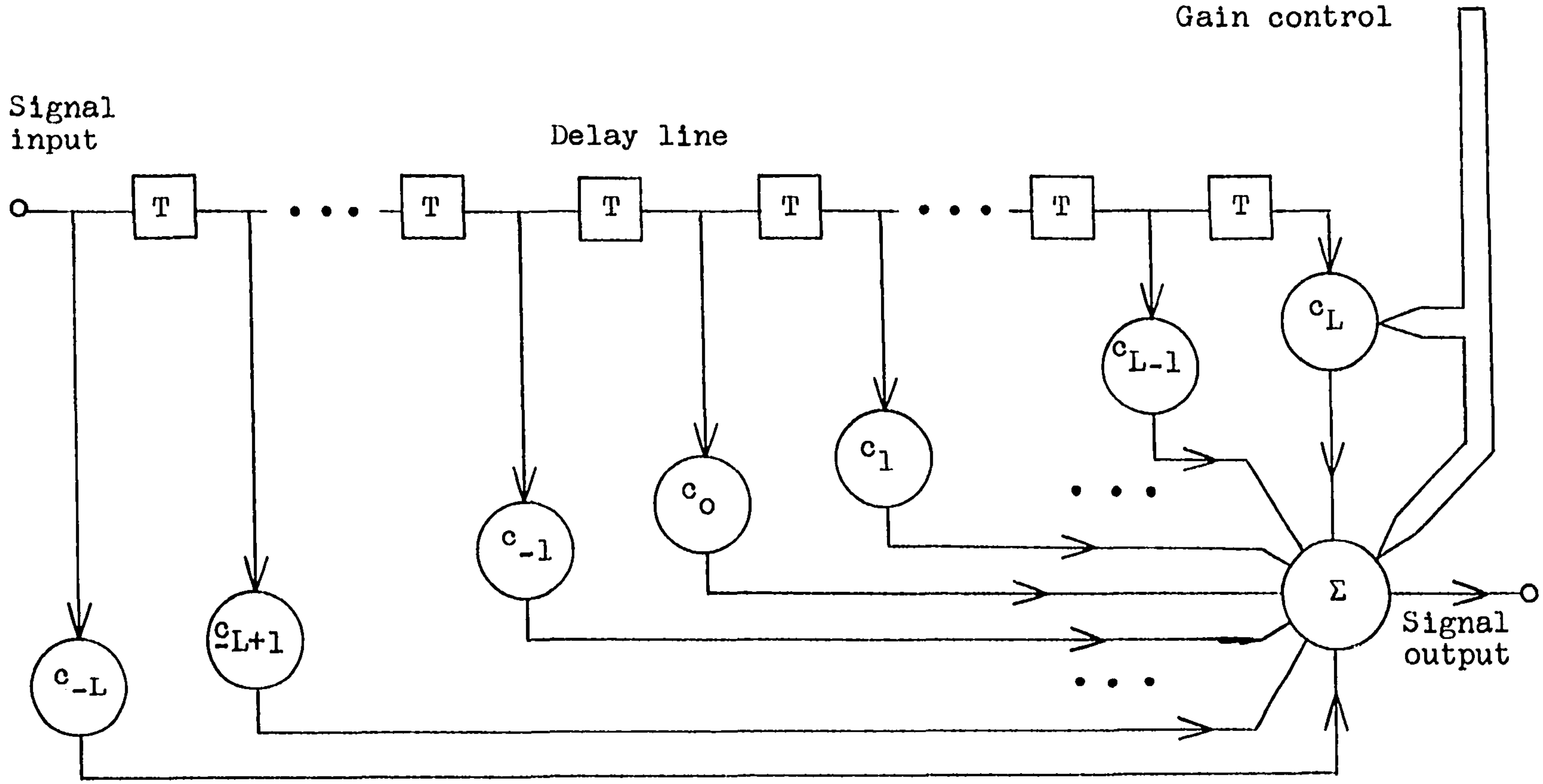


Figure 2. Block diagram of tapped delay line equalizer

the 60-80 MHz band, observing the output on an oscilloscope to adjust the directional-coupler tap gains. In a data set application Gibson [17] made adjustments by observing the receiver "eye pattern" [5], thus minimizing intersymbol interference. Gibson points out that the transversal filter can, in theory, be used to synthesize any amplitude and phase vs. frequency characteristic. However, no algorithms for implementation are presented. Kisel [23] analyzes a phase-correcting system consisting of a delay line in the feedback loop of a summator.

In time-domain equalizer terminology, peak distortion is defined as the normalized sum of unwanted pulse contribution magnitudes, measured at the sample time. Also in this context, mean-square distortion is defined as the normalized sum of squared unwanted pulse contributions. The minimization of either is usually accomplished by iteratively adjusting the tap gains via hardware feedback control. The term "automatic equalization" usually implies preset equalization, with adjustments occurring during a training period, after which the control mechanism is disabled and tap gains are "frozen".

There are many disclosures of automated transversal filter means for minimizing the time-domain intersymbol interference when synchronous (fixed, known pulse repetition rate) data is transmitted. Some of these are Rappeport [37], Coll and George [8], Becker et al. [2], Schreiner et al. [41], Tufts [44], Lucky [27], Gorog [18], Gersho [15], Tong and Liu [43], and Niessen [34]. A tutorial discussion of automatic equalization is given by Rudin [39]. The contribution of many of these authors is in the area of time-domain adjustment algorithms, with careful attention to convergence properties, since an iterative process in the presence of noise is employed. For example, the mean-square distortion minimization is based upon nulling the cross-correlation between the error and the input pulse.

Recent developments in equalization for synchronous data transmission have lead to "adaptive equalizers". Changes in the channel characteristic result in the equalizer control adapting the settings to maintain minimum distortion. Performance considerations are apparent in the articles by Lucky [28], Niessen and Drouilhet [35], DiToro [11], Gersho [16], Dominiak and Pickholtz [12], Hirsch and Wolf [20], Niessen and Willim [36], Lender [26], De and Davies [10], and George <u>et</u> <u>al</u>. [14]. Some of the devices are incorporated into the data receiver and utilize digital equalization circuitry (shift registers), since the pulse duration is known and constant. Synchronous data adaptive equalization allows optimum performance, although the hardware configuration is dictated by the data format and hence is more likely to be employed in the channel user's data set than in the common carrier's equipment.

The dominance of the channel duty cycle over that of the data set may justify equalization capability in the transmission facilities only, due to economic considerations. Furthermore, the facilities can thus be equalized for all users of the channel. Considerations such as these have lead to automatic and adaptive equalizers for general-purpose communication channels. These are described by Lucky and Rudin [29], [30], [40]. In both the automatic and adaptive versions the distortion to be minimized is the weighted squared magnitude of the difference between the equalized channel frequency-domain characteristic and that of an ideal channel, integrated over the frequency range of interest. The analysis is conducted in the time domain, using Parseval's theorem [31] to obtain an expression for the distortion in terms of the time functions corresponding to the several frequency characteristics mentioned above.

The implementation is based upon the transmission of repetitive pulses during the preset equalizer training period. The hardware is designed to iteratively adjust the various tap gains in the direction to minimize the cross-correlation between the error samples and the tap point signals. In the adaptive model the principle is the same except that the pulses must be of very low level to avoid interference with the user's data transmission. Hence a psuedorandom sequence having a spiked, periodic autocorrelation function is employed as the test signal. Since the information signal appears as a high level noise, the settling time of the adaptive equalizer may be several tens of seconds.

The automatic equalizer system model reported here is also intended for general-purpose communication channels. The concept of operation differs from the generalized equalizers noted above in several respects. It is assumed that the channel characteristic is automatically measured at uniformly spaced frequencies in the passband before release to the user at each data call. This is made feasible by the recent development of a fast and accurate automatic measurement device [38]. The device measures several critical channel parameters, allowing faulty channels to be flagged before customer transmission is attempted. The fact that the frequency-domain measurements obtained may be useful for determining the settings of an equalizer has stimulated this investigation.

The number of equalizer delay sections required to achieve an acceptable level of distortion is a variable from one channel to another. Hence it seems practical to connect together only the required number from an array of sections mutually accessible to several terminating channels at a central installation, permitting economy in control, signal, and computational hardware. The equalized channel characteristic is computed before the equalizer is "constructed"--if desired, the actual characteristic may be rapidly measured after set-up, to provide a record of the channel before release to the user.

# III. METHOD OF PROCEDURE

The frequency-domain characteristic of a channel is given by its amplitude response and phase shift vs. frequency [5]. It is the Fourier transform of the channel impulse response. The following notation is defined for subsequent use:

The underbar indicates a complex quantity. The overbar indicates a vector.

$w$ = radian frequency, radians/second.

$j = \sqrt{-1}$

The characteristic of the "raw" channel is

$\underline{X}(w) = |\underline{X}(w)| \cdot \exp[-j\ \theta(w)] = X(w) \cdot [\cos\theta(w) - j \sin\theta(w)]$.

It is comprised of a real amplitude response, $X(w) = |X(w)|$, and a real phase shift, $\theta(w)$.

The characteristic of the equalizer is

$\underline{C}(w) = |\underline{C}(w)| \cdot \exp[-j\ \phi(w)] = C(w) \cdot [\cos\phi(w) - j \sin\phi(w)]$.

The characteristic of the equalized channel is

$\underline{H}(w) = \underline{X}(w) \cdot \underline{C}(w) = X(w) \cdot C(w) \cdot \exp[-j(\theta(w) + \phi(w))]$.

The characteristic of the ideal channel is

$\underline{G}(w) = |\underline{G}(w)| \cdot \exp[-j\ q(w)] = G(w) \cdot [\cos q(w) - j \sin q(w)]$.

According to this notation if the input voltage to the channel is given by $V_{in} = \cos wt$, the output voltage is $V_{out} = X(w) . \cos[wt - \theta(w)]$. Note that a positive value of the characteristic phase shift function (e.g., $\theta(w)$ above) denotes a lag of the output phase with respect to the input at the particular frequency w. This notation is a standard one for the circuits/systems being considered here.

Usually the phase shift is not empirically determined directly; instead, the differential group envelope delay vs. frequency is measured [4]. From this the phase shift function can be derived to within an arbitrary reference slope and intercept. The absolute envelope delay of the channel at $w_o$ is given by the tangent slope of the phase shift, that is,

$$\text{absolute envelope delay} = \left.\frac{d\,\theta(w)}{d\,w}\right|_{w_o} .$$

It is the amount of time delay encountered by a narrow-band modulation envelope transmitted by the carrier $w_o$ through the channel.

Absolute envelope delay is usually not measured and is typically not critical in data transmission [5]. The crucial parameter (and the one specified and measured) is the differential envelope delay, that is, the delay at a particular frequency $w_o$ relative to the delay at some reference frequency $w_R$. Thus mathematically,

$$\text{differential envelope delay}\Big|_{w_o} = \left.\frac{d\,\theta(w)}{d\,w}\right|_{w_o} - \left.\frac{d\,\theta(w)}{d\,w}\right|_{w_R} .$$

In practice only the difference in delay as a function of frequency is measured by the test equipment, and for this reason the absolute delay is not known. When the minimum differential envelope delay is assigned a value of zero, other values define the nonnegative differential envelope delay function considered here. The phase is unknown in slope and in zero-frequency intercept. It is assumed here, without loss of generality, that both the raw channel and the ideal channel possess a phase intercept of zero.

It is thus possible to reconstruct a meaningful simulation of the channel phase shift function by integrating the differential envelope delay across the specified bandwidth. In the portion of the computer program which simulates the channel characteristic, numerical integration is used since delay values are given at only a limited number of frequencies. The channels considered are essentially low pass; the delay value at the lowest specified frequency is assumed to apply arbitrarily close to zero frequency as well.

With these definitions of the channel characteristic in mind one can specify the maximum allowable variation from a reference for various segments in the bandwidth in terms of amplitude response at 1000 Hz; which in turn is called the loss or gain. This latter specification varies with the particular installation/facility; it is assumed here that zero dB loss is desired. Figure 3 shows the requirements of Interstate Tariff FCC No. 260 for C4 grade circuit designation, suitable for transmission at 4800 bits/second or higher. The delay specification has been modified to be symmetrical about zero delay for the purposes of this investigation. Such requirements are meaningful in terms of potential data transmission performance and for governing the conditioning

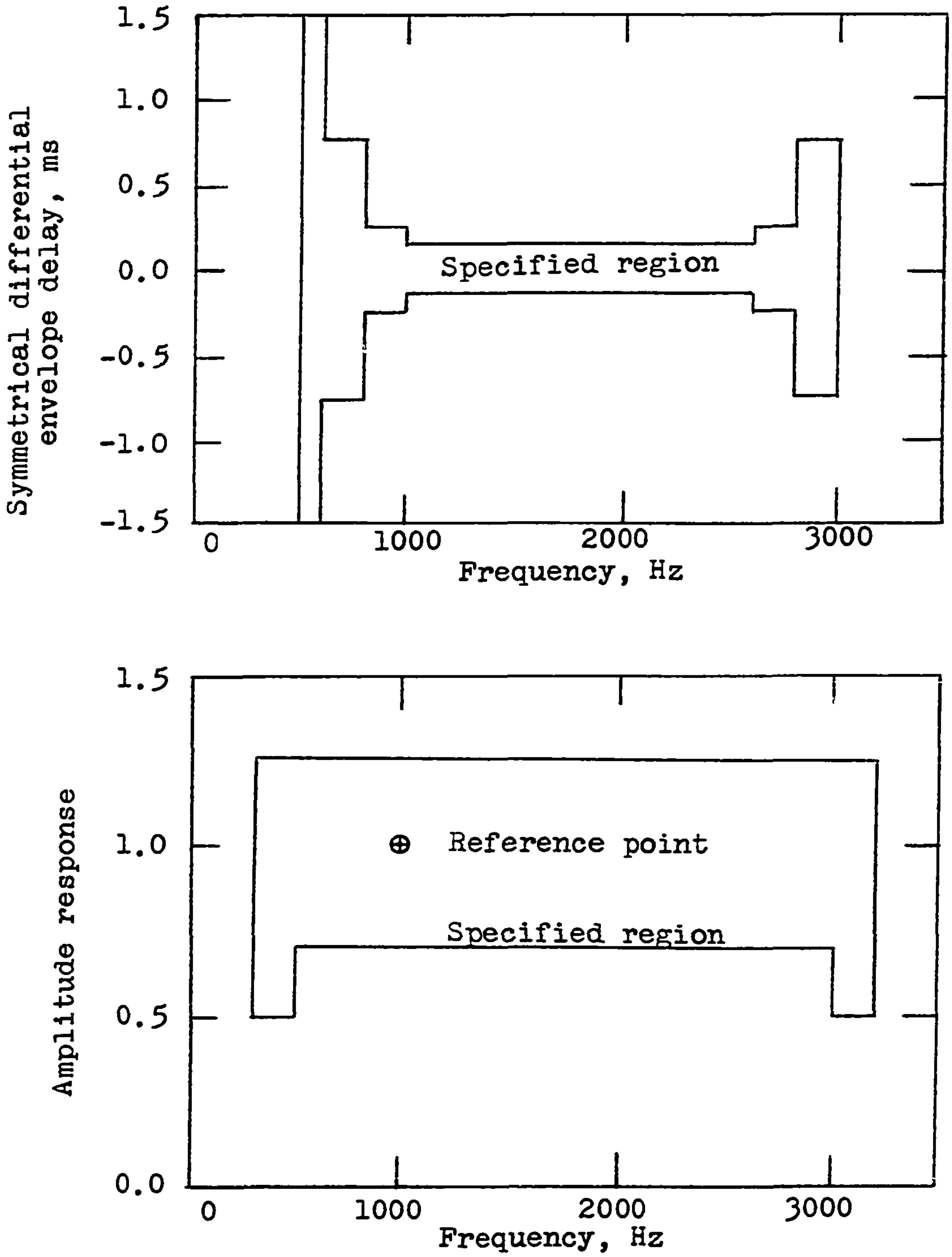


Figure 3. C4 grade specifications

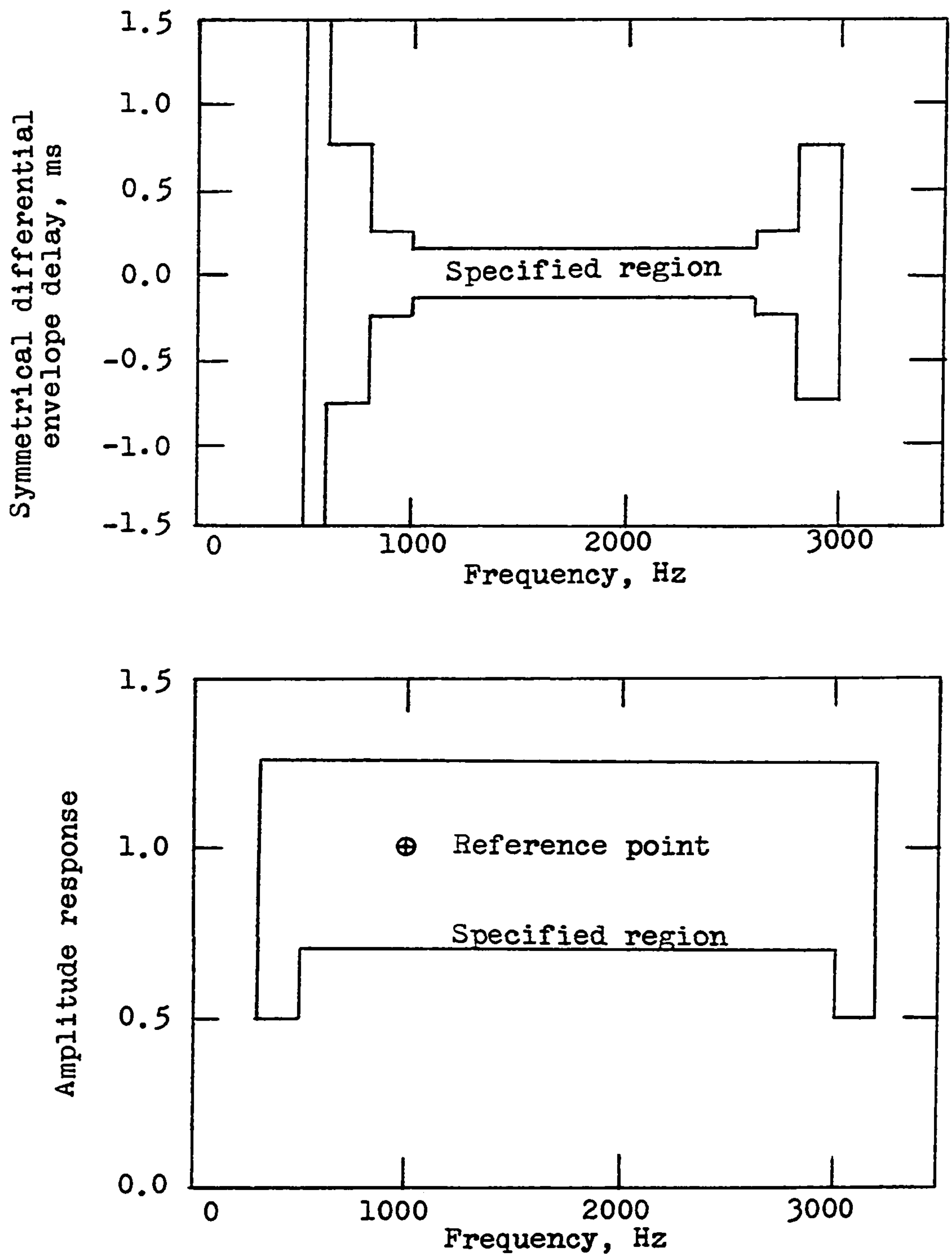


**Figure 3. C4 grade specifications**

of leased lines.

The frequency-domain error criterion for the equalized channel [30] is used in this investigation; it is defined by

$$E = \int_{-\infty}^{\infty} \left| \underline{H}(w) - \underline{G}(w) \right|^2 \cdot W^2(w) \cdot dw,$$

where $W^2(w)$ is a real nonnegative weighting function which assigns a relative weight to the equalization error at each frequency. In developing the equalizer algorithm one is at liberty to choose the error weighting function to aid in accomplishing the defined goal. When the frequency spectrum of the channel input signal is known it is possible to form the weighting function in such a manner as to achieve optimum transmission performance. However, as considered here, the channel is to be suitable for generalized usage and to meet specifications like those shown in Figure 3. Hence weighting functions are chosen which assign the highest "priority" on equalization to frequency segments whose tolerances in amplitude response and differential delay are the smallest. Weightings numbered 1 and 2 shown in Figure 4 are examples used for this purpose.

To derive the algorithm assume initially that the equalizer summing amplifier gain, g, is fixed at unity and that the various tap gains, $c_n$, are not limited in setting range or accuracy. Let the time reference be $\frac{1}{2}(N-1)\cdot T$ seconds, the delay in passing from the input terminal to the midpoint

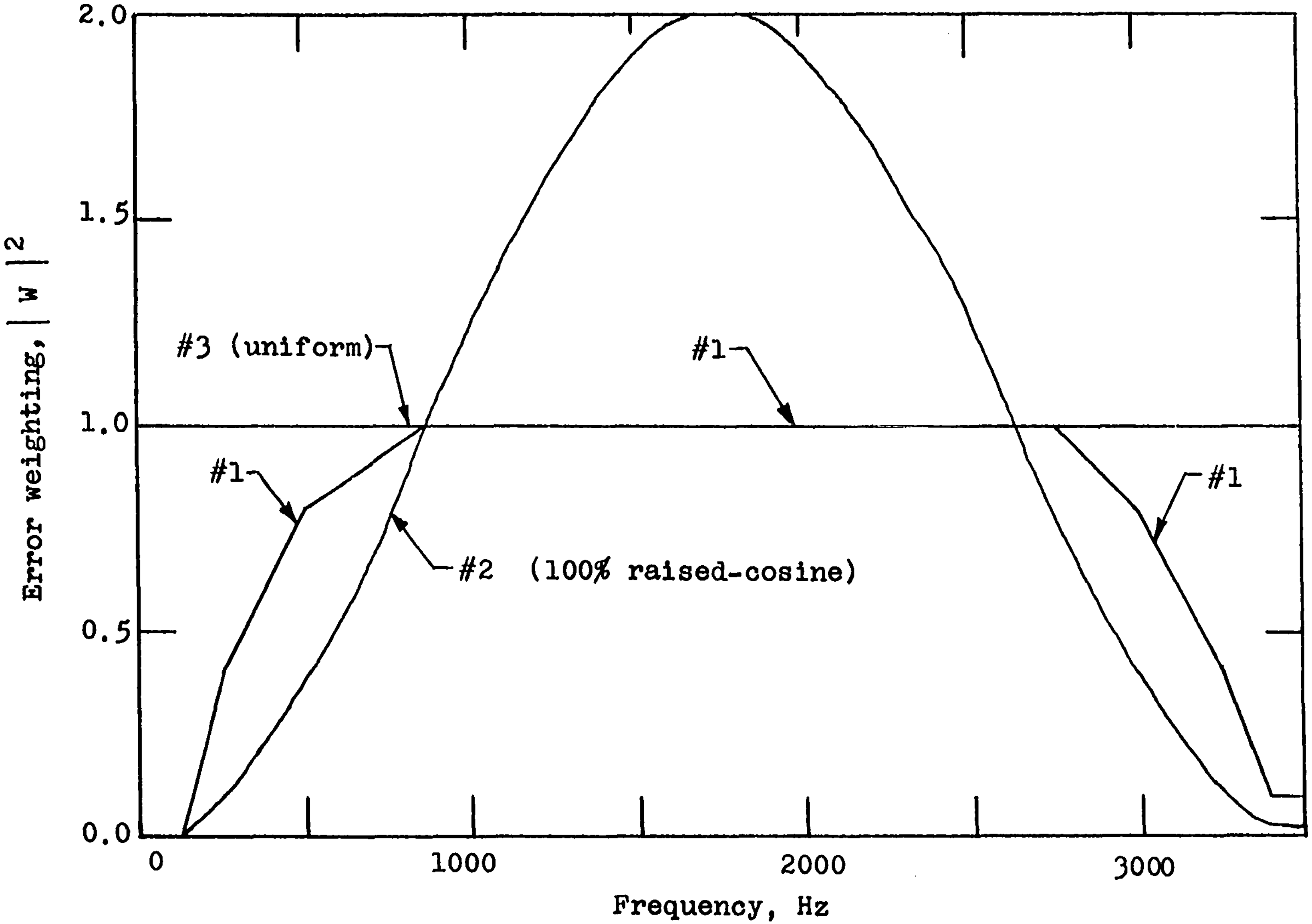


Figure 4. Error weighting functions

of the equalizer. Recall that $N=2L+1$ is the total number of tap points in the equalizer (Figure 2) and $T$ is the delay per section (between tap points) of the line. It is assumed here that the phase shift of each section is a linear function of frequency with constant slope $T$ over a frequency range significantly greater than the bandwidth being considered. This assumption is valid for bandwidths on the order of $(10/T)$ Hz or less [33].

The impulse response of the line at the nth tap point from the midpoint is $\delta(t-nT)$. Here $\delta(u)$ is the Dirac delta function (an impulse of time width zero and area under the impulse of unity) occurring at $u=0$. According to the time reference designation, n has positive integer values for tap points on the output side of the reference point (equalizer midpoint), and n has negative integer values for tap points on the input side. Each tap point signal is modified in amplitude by the respective tap gain factor, and then all such voltages are summed to form the output. Hence the tapped delay line impulse response is

$$
\begin{aligned}
c(t) = {} & c_{-L}\cdot\delta[t - (-L)\cdot T] + c_{-L+1}\cdot\delta[t - (-L+1)\cdot T] + \ldots \\
& + c_0\cdot\delta(t) + \ldots \\
& + c_{L-1}\cdot\delta[t - (L-1)\cdot T] + c_L\cdot\delta[t-(L)\cdot T].
\end{aligned}
$$

The Fourier transform of $c(t)$ yields the frequency characteristic of the equalizer:

$$\underline{C}(w) = \sum_{n=-L}^{L} c_n \cdot \exp(-j\ wnT).$$

This is clearly a function only of the tap gain factors, $c_n$, given fixed values for T and N. The tapped delay line equalizer offers independent control of the amplitude- and phase- frequency characteristic only over a radian frequency range of $\pi/T$ radians/sec, due to the periodicity of the characteristic [30].

To develop the frequency-domain algorithm the error expression is now rewritten as

$$E = \int_{-\infty}^{\infty} \left| \underline{X}(w) \cdot \sum_{n=-L}^{L} c_n \cdot \exp(-j\ wnT) - \underline{G}(w) \right|^2 \cdot W^2(w) \cdot dw,$$

by direct substitution. If this error is differentiated with respect to each $c_k$ in turn and each resulting expression set equal to zero (for minimization of the error) a set of N linear algebraic equations in the $c_k$'s results. This is carried out in Appendix A. The resulting system of equations involving numerical integration is expressed in matrix notation by

$$A\ \bar{c} = \bar{b}\ .$$

Here A is a NxN coefficient matrix with elements of the form

$$a_{k,n} = \sum_{i=1}^{M} X^2(w_i) \cdot W^2(w_i) \cdot \cos[(n-k) \cdot w_i \cdot T]\ ,$$

$\bar{c}$ is a Nx1 solution vector representing the desired tap gain factors, and $\bar{b}$ is a Nx1 constant vector with elements of the form

$$b_k = \sum_{i=1}^{M} X(w_i)\cdot G(w_i)\cdot W^2(w_i)\cdot\cos[q(w_i) - \theta(w_i) - kw_iT] \;.$$

Here k and n are integers: $-L,-L+1,\cdots,0,\ldots,L-1,L$. Recall that $W^2(w)$ is the error weighting function. The definitions of the raw channel and ideal channel characteristics are as given at the beginning of this chapter.

The elements are expressed in terms of a summation in frequency rather than in integral form; since, as previously mentioned, the characteristic data is available as measurement values only at selected frequencies. As indicated by the summation limits there are a total of M equally-spaced frequency locations considered.

To reduce the dimensionality of the overall study the ideal channel characteristic is specified as being that of constant zero attenuation and some linear phase function of frequency. Thus notationally,

$G(w_i) = 1.0$ and $q(w_i) = p\cdot w_i$ for all $i = 1,2,\cdots,M$ .

This implies that the ideal channel possesses a constant envelope delay, p, at all frequencies, that is, the differential delay is everywhere zero. Only this particular form of ideal channel is included in the study as it is the most suitable for generalized data transmission utilization, assuming no knowledge of the channel noise or data signal spectra. Consequently the computer program is written for this form.

The "perfectly" equalized channel thus possesses a total absolute envelope delay (from channel input terminals to the equalizer output terminals) equal to the sum of the following terms:

(1) the absolute delay of the raw channel (at the frequency of zero differential envelope delay),

(2) the absolute delay of the equalizer from its input to its midpoint, given previously as $\frac{1}{2}(N-1)\cdot T$, and

(3) the ideal channel constant envelope delay, p, defined above.

Since total absolute envelope delay is not specified for Interstate Tariff FCC No. 260 conditioned channels, terms (1) and (2) are not considered further in the study. It has been assumed that term (1) is not known. Term (2) is an integer multiple of T and is dictated by the length of the line. It has no bearing on the solution algorithm.

It seems likely that the degree of success achieved by the equalization process depends upon the choice of the ideal channel delay, p, term (3) above. Since no specification has been placed on p it may be treated as a variable along with the tap gain factors. However, to do so results in a system of nonlinear equations for the condition of minimal error. Fortunately the problem is circumvented when the number of tap points utilized is more than sufficient to achieve acceptable equalization results. For then the tap gains far removed from the equalizer midpoint are assigned negligible values. As p is increased (or decreased) the optimum tap gains are modified. When p has been changed by exactly T seconds, the tap gain solutions are the same, except shifted by one subscript. The equalized channel error is thus a periodic function of p over a reasonable range of values. Thus if the initial value of p is sensibly chosen it is possible to obtain a near optimum ideal channel delay by iterating the value of p in one direction until a local minimum in the residual error is located. Such a technique is employed in this study, as indicated in the description of the digital computer simulation program which follows.

The method of procedure in the simulation is to utilize the algorithm (summarized by the equation, $A\bar{c} = \bar{b}$) to solve for the optimum tap gain settings. The amplitude response and phase shift function of the equalizer are then computed at specified locations in frequency. This information is combined with the raw channel characteristic to determine that of the equalized channel. The error is evaluated by comparing the resultant against the ideal channel. The equalized channel amplitude, phase and differential delay (obtained by finite differentiation) functions of frequency are printed, and a check is made against the C4 specifications to note exceptions.

One of the parameters of this investigation is the resolution of the tap gain settings. Quantizing the gain settings is necessary in implementation of digital control. Experience disclosed in the literature [31] indicates that in some applications the resolution is the limiting parameter in equalizer performance. The program provides for specification of the resolution as a parameter.

It is assumed that each equalizer section has perfectly matched input and output impedances and possesses unity gain amplitude response over all frequencies of interest. In practice amplifiers would be employed after each section to overcome insertion loss. It is also assumed that the taps introduce negligible loading on any section, due to the use of isolation amplifiers. The delay of each section is assumed to be a constant in frequency but provision has been made in the program to simulate a random variation (in an ensemble sense) in the delay per section about a specified mean value. The Monte Carlo technique [19] is used to accumulate statistics on the effect of the delay manufacturing tolerance by repeatedly assigning random values to the individual delays and computing the resultant equalized channel error as described above. No previous studies of this type have been noted in the literature. The delay is not likely adjustable in the field, whereas the various compensation and isolation amplifiers can be kept in correct operation by proper maintenance.

The following notation for parameters and variables is applicable to the simulation method:

N: number of tap points in the equalizer (always odd)

T: delay per section of the equalizer, seconds

F: frequency spacing between characteristic data points, Hz

M: number of frequency locations at which data is given

X(I): raw channel amplitude response at I-th frequency

D(I): raw channel differential delay at I-th frequency, seconds

W(I): error weighting function at I-th frequency

RES: resolution of tap gain settings

KVAR: number of delay-per-section variation sub-runs

AM: delay-per-section distribution mean (using AM=1.0 effectively centers the distribution on T)

SD: delay-per-section distribution standard deviation

REJ: rejection value for delay per section

Z: a random number (computer generated)

p: ideal channel delay, seconds (computed)

$c_n$: n-th tap gain setting (computed).

There are two modes available through an option in the program. The investigation mode is used to obtain simulation results for values of N over a specified interval. The operational mode is used to simulate the ability of the system to determine the minimum required number of tap points, thus demonstrating the utility of the technique.

The digital computer simulation method begins with the input of the various parameters and the raw channel amplitude and delay data. As previously described, the raw channel phase shift function is derived from the delay data using numerical integration. The ideal channel delay is initially set equal to a value which gives the same phase shift as the raw channel at the lowest frequency of zero differential delay as determined from the input data. A weighted phase error sum is formed by numerically integrating (in frequency) the weighted difference between the raw channel phase and the ideal phase. The value of p is then increased or decreased in successive iterations until the absolute value of the phase-error-sum is less than a prescribed tolerance.

The unequalized channel error using this value of p as the ideal channel delay is calculated and labeled the R-slope error. In the operational mode the number of tap points needed to achieve C4 specification compliance by the equalized channel is estimated, based upon a staircase function of the R-slope error. This determines the starting number of tap points considered in the operational mode. In the investigation mode p is increased by 20 percent to define the starting value which will be used in the search for a local minimum error. Such a search is not conducted in the operational mode.

The next step in the method is to compute the coefficient matrix and constant vector elements and then to solve the system of equations, $A\bar{c}=\bar{b}$, for the tap gains, using the selected value of p as the ideal channel delay. The program then proceeds with the equalizer characteristic computation and the determination of the equalized channel error. In the investigation mode the value of p is next reduced by 2.5% and the solution-evaluation procedure repeated until the equalized channel error has passed through a local minimum. In either mode, the value of p finally chosen is used in the remainder of the simulation for the run (set of parameters).

The characteristic of the equalizer is evaluated at "sub-increment" frequencies defined as integer multiples of F/8 for $F \geq 200$ Hz and F/4 for $F < 200$ Hz. This technique eliminates the "branch" ambiguity of arctangent function used in computing the equalizer phase since the phase is not expected to change more than $\pi/2$ radians over these intervals. To insure proper tracking near zero frequency the first sub-increment is divided into 8 fine-frequency increments. The equalized channel error is computed at integer multiples of F/2 in all situations. The raw channel characteristic at each odd multiple of F/2 is estimated by linear interpolation.

In the above characteristic evaluating procedure the summing amplifier gain is defined by the largest tap gain absolute value. The tap gains are normalized so that the maximum absolute value is 1.0 with appropriate signs retained. The resolution of the tap and summing amplifier gains is initially defined as .0001, so the gains are quantized using this value.

Next the system performance is determined for the tap gain resolution set equal to the parameter RES. The tap gain solutions obtained originally are used--only the quantizing levels are changed.

Subsequently the random variation of the equalizer delays is introduced, with each section delay set equal to Z·T. Here Z is obtained from a random number generator subroutine which gives a normal distribution with standard deviation, SD, about the mean, AM. The power-residue method of generating uniformly distributed random numbers [19] is employed in subroutine RANDU [21]. Twelve such numbers are summed in subroutine GAUSS [21] to generate the normally distributed random number, Z, using the method described by Hemmerle [19].

If $|Z - AM| >$ REJ the value of Z is rejected, and a new one obtained. Thus the resultant distribution is a truncated normal distribution with standard deviation somewhat less than originally specified. This closely models a manufacturing quality control procedure. The system performance is calculated using the perturbated delay line mode. This entire Monte Carlo process is repeated the number of times specified by KVAR.

Upon completion of the sub-runs, in the investigation mode the delay line model is reset to the perfect condition, and the resolution is redefined as 10·RES and subsequently as 40·RES. The system performance is calculated for each tap gain resolution.

This completes the simulation for the run. The program proceeds immediately to calculation of tap gains and equalizer performance for an increased number of tap points, if so directed. In the investigation mode this occurs if the prescribed interval of N has not been covered. In the operational mode N is increased until C4 specifications have been met by the equalized channel.

Several details of the system model are omitted in this description of the procedure and are given special attention in the discussion of findings (Chapter IV). The details of the simulation program are apparent in the program flow chart and listing (Appendix C).

The algorithm derived in Appendix A and presented in this chapter could be used equally well in "filter" synthesis with the tapped delay line considered as the filter. In this situation it would be convenient to assign values to the "nonexistent raw channel" as follows:

$X(w_i) = 1.0$ and $\theta(w_i) = 0.0$ for all $i = 1,2,\cdots,M$.

The desired filter characteristic would be specified by means of the ideal amplitude response $G(w_i)$ and phase shift function $q(w_i)$. The error weighting function $W^2(w_i)$ would be chosen by the designer based upon the synthesis accuracy requirements as a function of frequency. Such an application would require some modifications to the program presented here.

# IV. DISCUSSION OF FINDINGS

This chapter reports the findings on the equalization algorithm and system model performance for various parameters and sample distortion cases. Some of the method details which are based upon auxiliary investigations using special forms of the program are also reported.

The raw channel characteristics utilized are fabricated to represent practical situations and at the same time be suitable for "single-number representation". To have a basis for comparison with results reported by Lucky _et_ _al_. [31] for time-domain mean-square distortion equalization, a portion of the channel distortion cases are specified by the format shown in Figure 5. The frequency range chosen in these investigations is 0 to 3500 Hz. The range chosen for some special cases is 0 to 3200 Hz. Table 1 lists all of the distortion cases reported here.

The differential envelope delay of cases 1-4 is a parabolic function of frequency as shown in Figure 5. Placing the parabola axis at 1750 Hz gives a symmetric delay function in the chosen bandwidth. The delay can be specified by a single number, the maximum delay at the band edge. To relate this to data transmission performance assume that the center frequency, 1750 Hz, is defined as the carrier and that double-sideband amplitude modulation is employed. For Nyquist criterion transmission techniques utilizing the full bandwidth

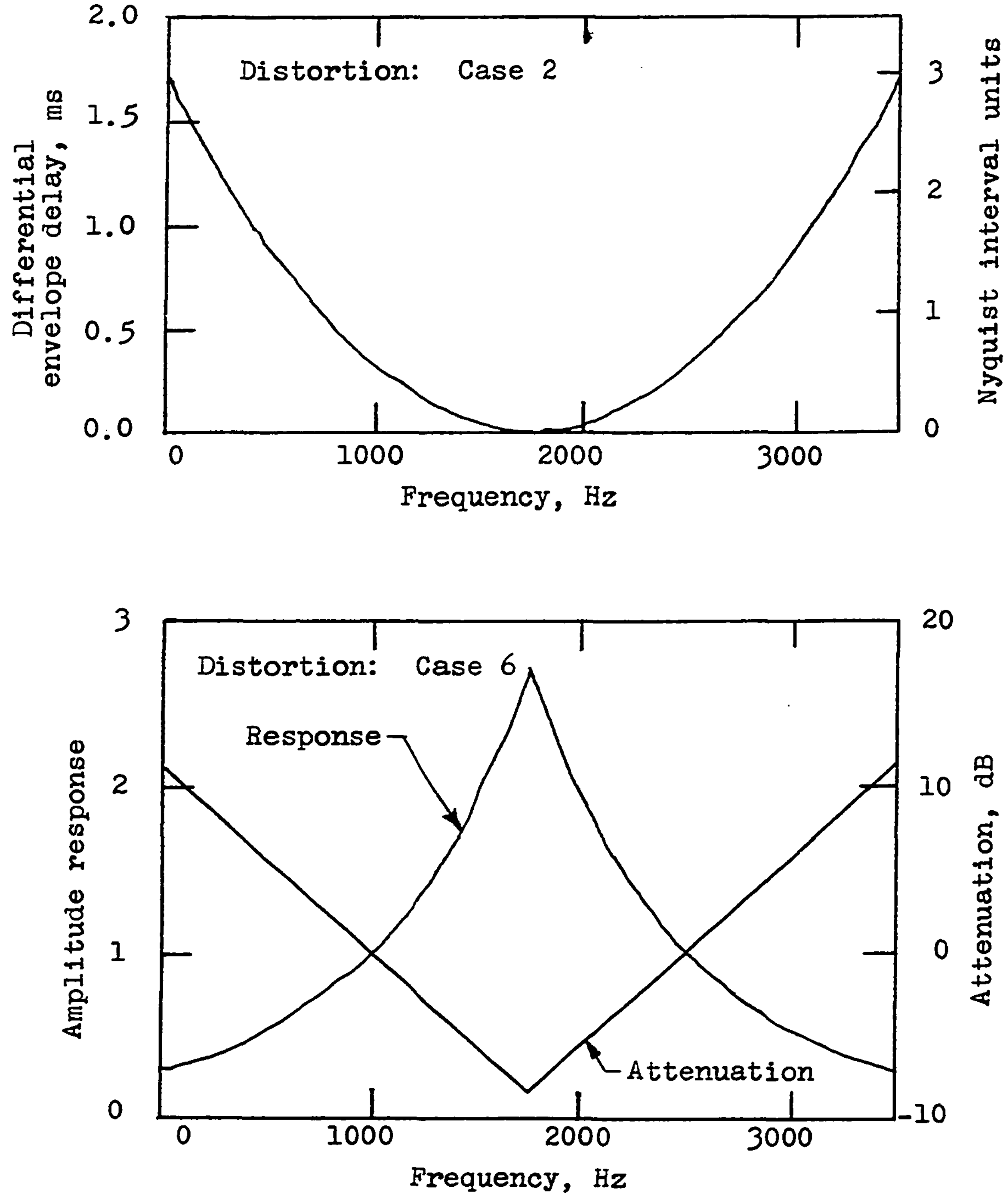


Figure 5. Linear dB attenuation and parabolic delay format, 3500 Hz bandwidth

(assuming a full raised-cosine pulse spectrum) the maximum signaling rate is $f_{max}$ = 1750 pulses/second. Thus the Nyquist interval is $1/f_{max}$ or .572 ms. Transmission impairments reported in the literature are sometimes related to the maximum delay expressed in Nyquist interval units [42]. For example, 1.72 ms maximum delay is three Nyquist interval units for the 3500 Hz passband considered.

Table 1. Channel distortion cases

| Case number | Figure number | Delay distortion | Attenuation distortion |
|---|---|---|---|
| 1 | | parabolic, 0.86 ms max. | none |
| 2 | 5 | parabolic, 1.72 ms max. | none |
| 3 | | parabolic, 2.29 ms max. | none |
| 4 | | parabolic, 2.86 ms max. | none |
| 5 | | none | linear dB, 10 dB max. |
| 6 | 5 | none | linear dB, 20 dB max. |
| 7 | | none | linear dB, 30 dB max. |
| 8 | 16 | case 2 | case 5 |
| 9 | 8 | irreg. parabolic, 3 ms max. | approx. lin. dB, 25 dB max. |
| 10 | 9 | irreg. parabolic, 3.2 ms max. | irreg. lin. dB, 18 dB max. |
| 11 | 12 | none | periodic components |
| 12 | 13 | periodic components | periodic components |
| 13 | 17 | irregular | irregular |

Similarily, the amplitude distortion is represented as a linear attenuation (in dB) increasing from the band center to the band edges so that it can be specified by a single number, the difference between the minimum and the maximum attenuation expressed in decibels. The input measurement data is normalized so that the amplitude response is unity at 1000 Hz, in keeping with U.S. practice.

Although the computer program is generalized, most of the investigations reported are for F=250 Hz as the frequency spacing between channel characteristic measurement points. The value of 250 Hz is used in the previously-referenced, commercially-available automatic test equipment which provides data of the type required; this is the primary spacing studied here. The value, F=100 Hz, is used as a control for comparison purposes.

The first topic to be considered is the method of utilizing the channel data in determining the system matrix equation elements. The intuitive choice is to perform the numerical integration at exactly the measurement frequency points. This is referred to as the "point specification algorithm". It results in extremely good equalization at the mentioned frequencies (integer multiples of 250 Hz) given an adequate number of taps. However, as can be seen from Figure 6, the equalization may be extremely poor at the midpoint frequencies. The points indicated as "theoretical" represent the desired phase response of the equalizer for this parabolic delay channel. For the point algorithm the number of taps used is such that the phase is essentially perfectly equalized at the specified points. The residual error not considering midpoints is about .001 percent, but it is evident that the equalized channel is of little utility because of the wide

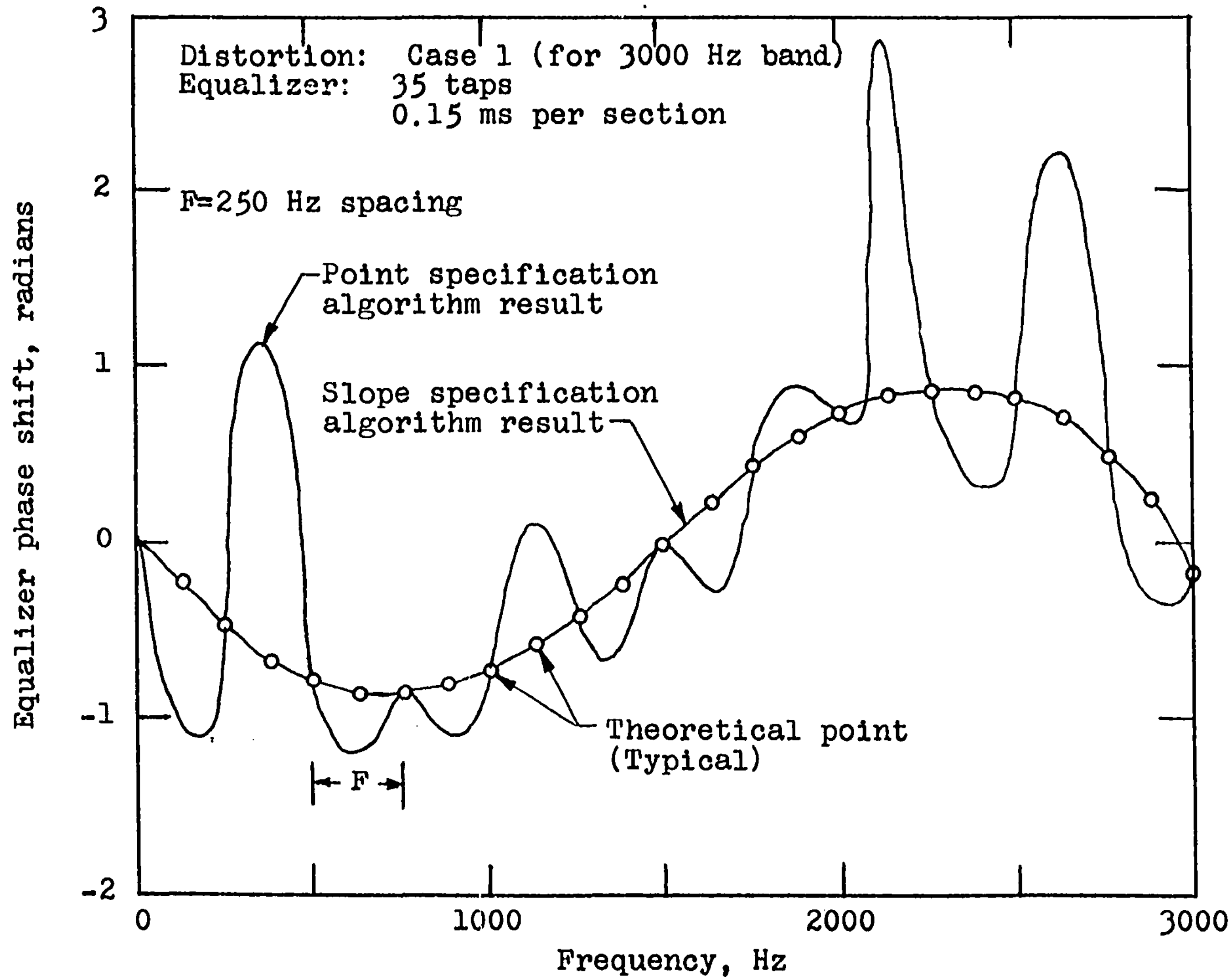


Figure 6. Equalizer phase shift function for two algorithms, case 1

excursions in phase. (The amplitude response is similarly erratic.)

To avoid equalization errors of this type the slope specification algorithm is used throughout this investigation. The numerical integration to determine the matrix equation elements uses linearily interpolated channel amplitude and phase data at frequency points $\pm$ F/8 Hz from each nominal frequency. Thus the term "slope specification algorithm" is used. This is a realistic approach since the characteristic is not likely to differ radically from the interpolated values for this small frequency change. Furthermore, the residual channel error is evaluated at the midpoint and at the nominal frequency to detect erratic performance. Figure 6 also shows the result obtained using this algorithm. The residual error is on the order of 5%, although this result has been obtained without the benefit of further optimum choices to be described below. This equalized channel is very suitable for data transmission, as its amplitude response is similarly near-ideal.

The equalization error, E, is obtained by numerical integration over the 2·M equally spaced frequencies where the incremental weighted squared error term,

$e = \left| \underline{H}(w) - \underline{G}(w) \right|^2 \cdot W^2(w) \cdot dw,$

is formed. This represents the weighted, squared magnitude of the difference between the equalized and ideal channel characteristics. A weighting-area correction factor, $A_{eq}$, is defined as the weighting-function average over the bandwidth, BW. This allows all errors to be scaled for a unity-average weighting function. An error reference is defined as the expected error assuming an uniformly distributed phase error (0 to $2\pi$ radians) for the equalized channel. As shown in Appendix B, the equalized channel (rms) error used in these discussions as the performance index is given by the expression, $[E/(8\cdot\pi\cdot BW\cdot A_{eq})]^{\frac{1}{2}}$ .

For this scaling the unequalized channel errors for distortion cases 1-7 are very similar to those computed by Lucky et al. [31] using a time-domain mean-square error expression.

In Chapter III the "ideal channel delay" selection procedure is discussed. Figure 7 presents the variation in equalized channel error as a function of the ideal channel delay, p, for distortion cases 9 and 10, each having combined amplitude and phase distortion in the raw channel. The distortion characteristics are plotted in Figures 8 and 9. The results have been obtained using a special form of the program. The periodic variation for reasonable values of p is apparent, with the period being equal to the delay per section. The standard program selects the initial value of p such that the ideal channel phase function is a linear approximation to the raw channel phase function. Thus the (local-minimum error vs. p) search should achieve near

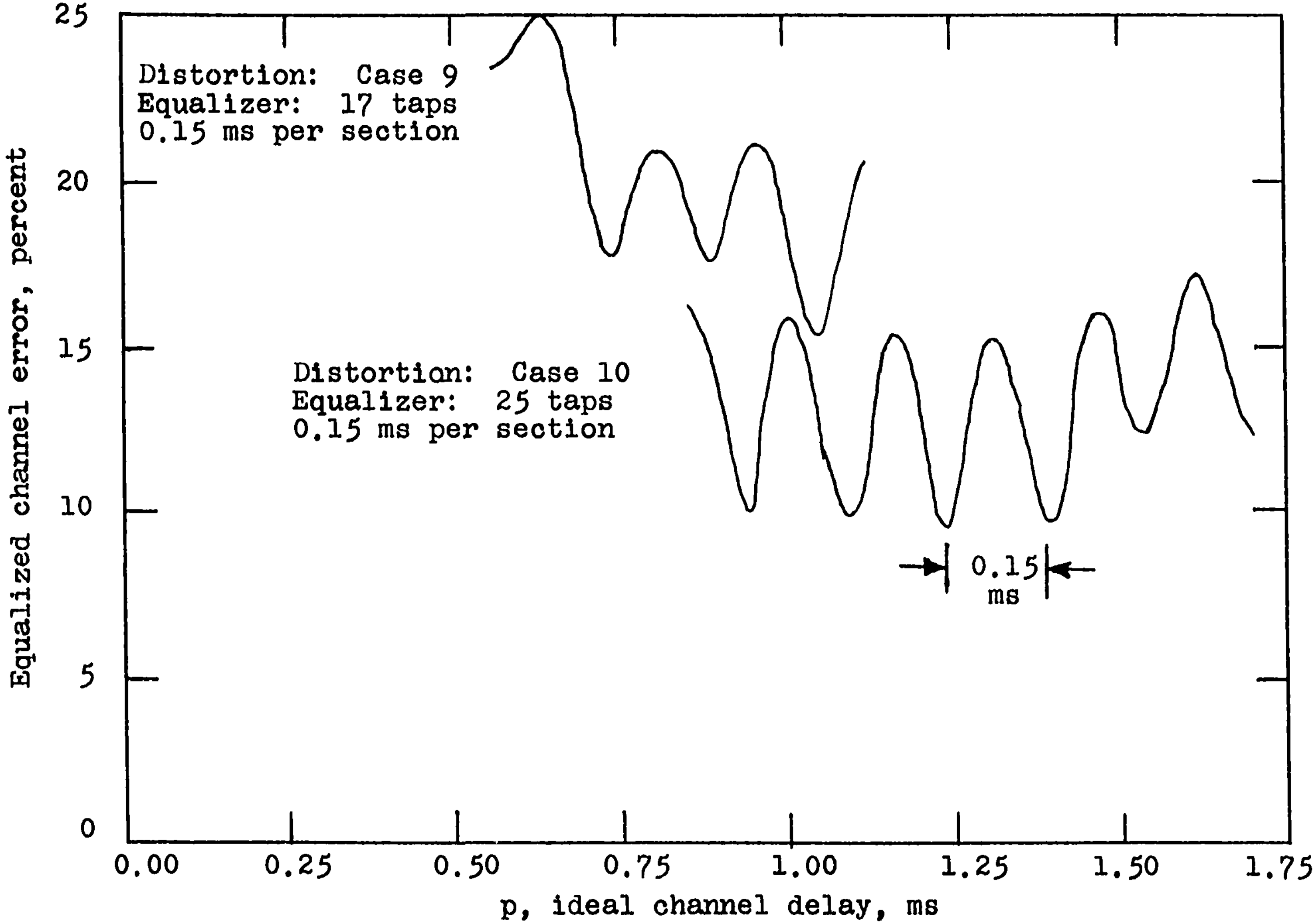


Figure 7. Equalized channel error vs. ideal channel delay, cases 9 and 10

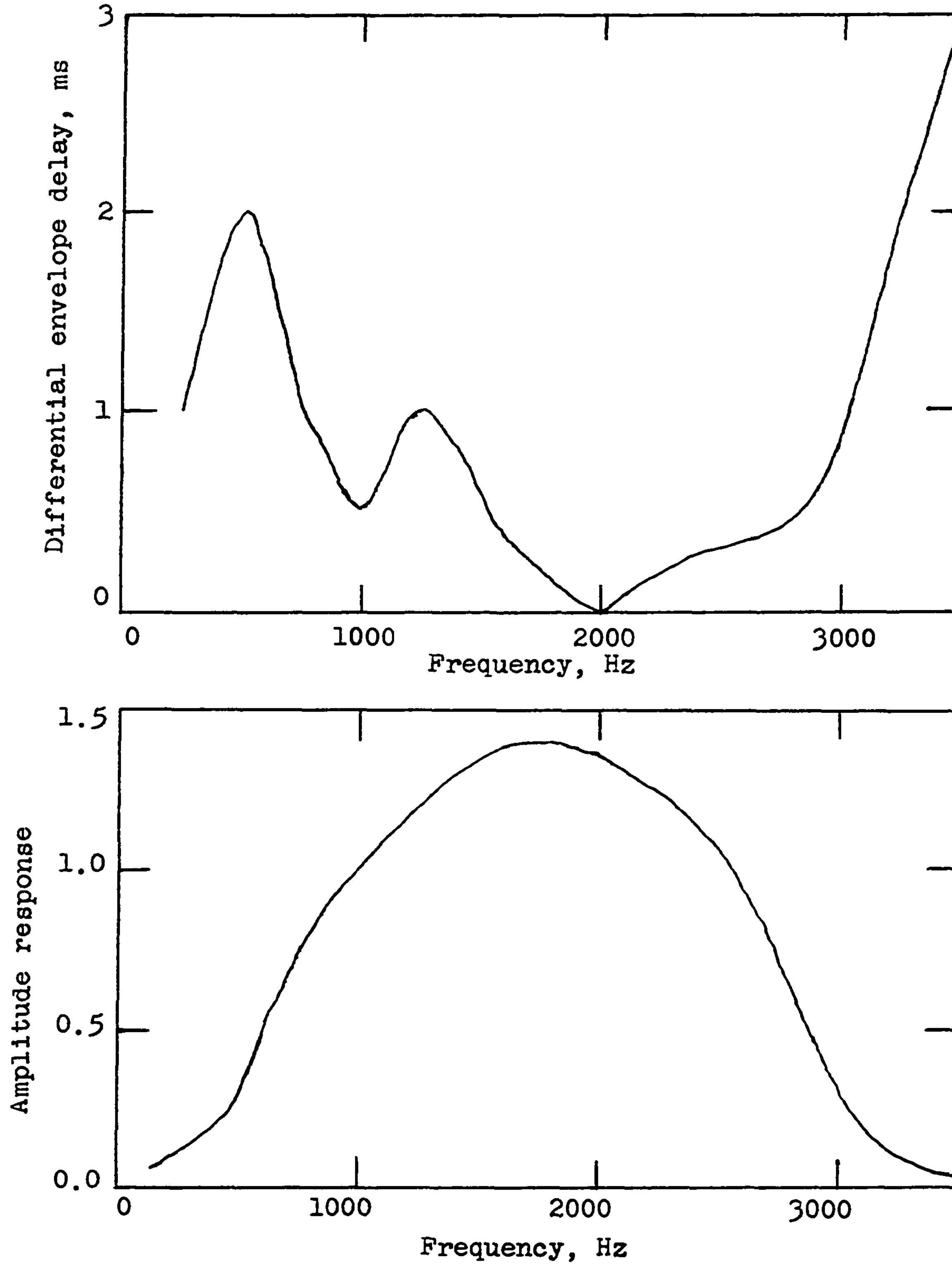


Figure 8. Case 9 characteristic

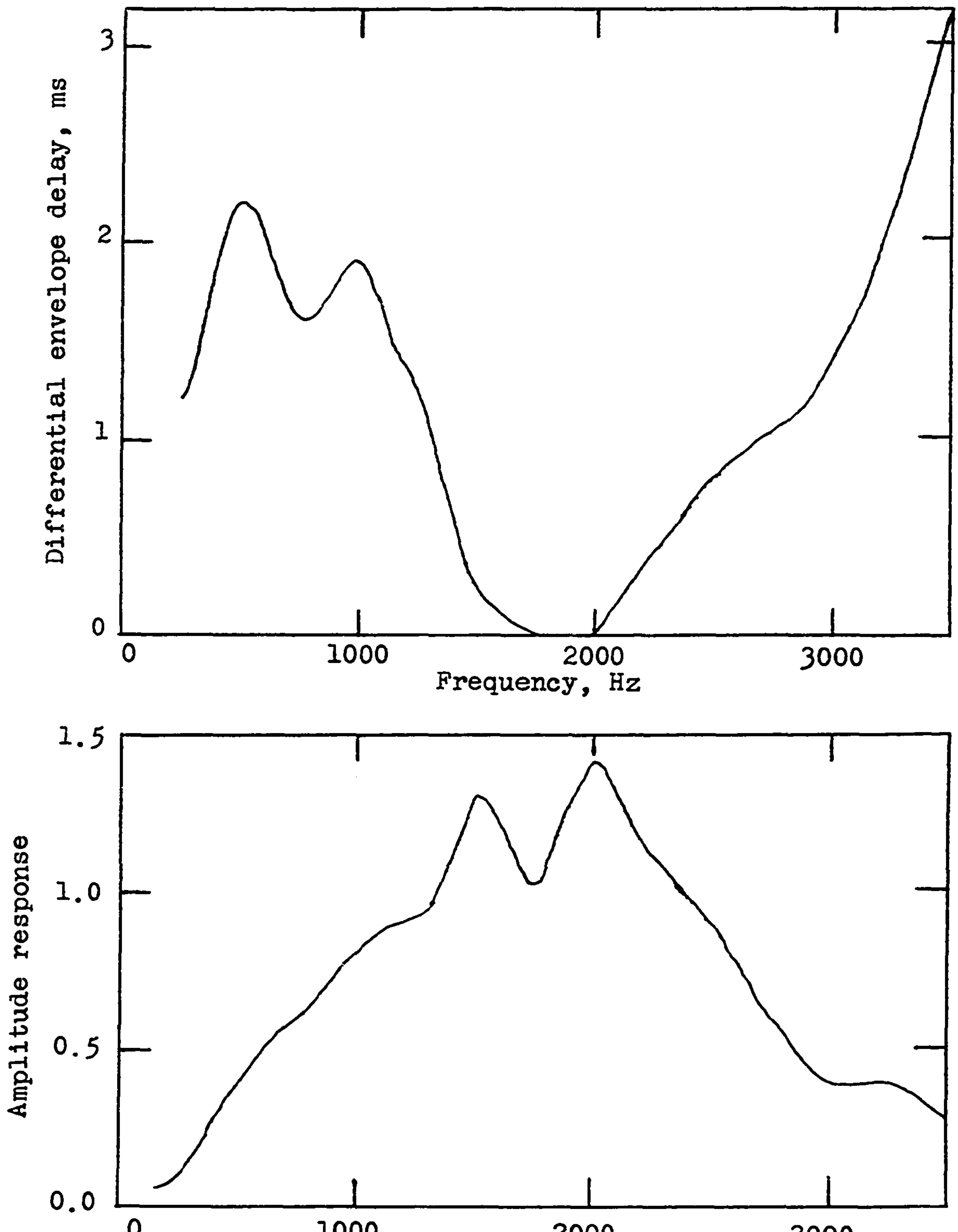


Figure 9. Case 10 characteristic

optimum ideal channel delay for the given number of tap points. If the raw channel has no differential delay the procedure will automatically set p=0, as desired.

The periodic variation in channel error with p has been noted on many delay distortion runs where the number of tap points and the delay per section are appropriate for the channel condition. A special form of data is presented for case 10. Table 2 lists computed tap gains for selected values of p, spaced exactly 2·T and then approximately 1·T apart. It is to be noted that the tap gain solution values shift further along the delay line as p is increased. The equalized channel error is essentially the same in each instance, corresponding to local minima of the periodic function mentioned above.

Figure 10 is a family of plots showing channel error vs. the ideal delay for a fixed parabolic delay condition (case 2) with the number of equalizer tap points as a parameter. Note the essentially sinusoidal dependence exhibited for each case and that the amplitude of the variation decreases as the "equalizing capability" (related to N) is increased. Again the apparent period is equal to the delay per section. All curves start on the right with an initial value of p near 0.55 ms, a number which yields the "summed phase error" within the specified tolerance from zero. This starting point is a function only of the raw channel distortion, not of the equalizer configuration. Note also that each curve terminates

Table 2. Computed tap gains for distortion case 10

| | Tap number[a] | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | -9 | -8 | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| $p_1$ | .18 | -.01 | .50 | .37 | .99 | .64 | 1.00 | .55 | .58 | .68 | .42 | .77 | .24 | .24 | .39 | .26 | .05 | .06 | .09 |
| $p_2$ | .08 | -.04 | .20 | .01 | .50 | .38 | .99 | .64 | 1.00 | .54 | .60 | .67 | .43 | .73 | .25 | .25 | .40 | .27 | .07 |
| $p_3$ | .06 | .14 | -.04 | .26 | .01 | .52 | .39 | 1.00 | .72 | .99 | .66 | .52 | .75 | .38 | .74 | .27 | .20 | .37 | .27 |

| p, ideal channel delay, ms | Equalized channel rms error, % |
|---|---|
| $p_1$ = 0.93 | 9.9 |
| $p_2$ = 1.23 | 9.5 |
| $p_3$ = 1.40 | 9.7 |

[a]For N = 25 taps. The gains for taps numbered ± 10, ±11, ± 12 are all less than or equal to .10 in magnitude and are not shown. Delay per section, T = 0.15 ms.

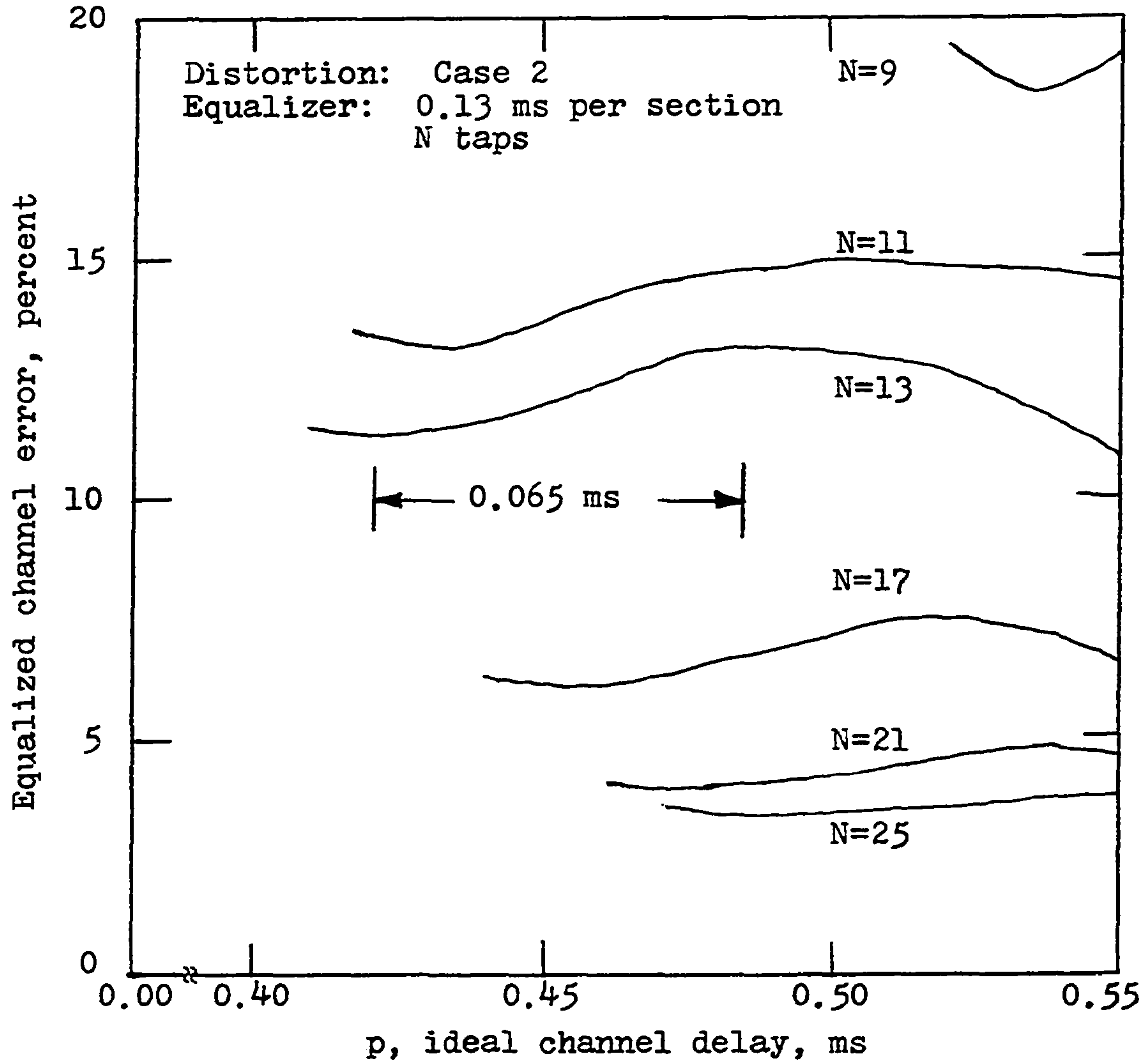


Figure 10. Equalized channel error vs. ideal channel delay case 2

one step after the local minimum has been identified, as p is decreased. These results have been obtained as a byproduct of investigation mode equalization runs. Because the amplitude of the error variation is small for N large enough to yield good results, the operational mode does not employ the local-minimum error search.

Attention is now directed to the choice of T, the delay per section. Consider the equalizer characteristic, which is given in Chapter III as

$$\underline{C}(w) = \sum_{n=-L}^{L} c_n \exp(-j\, wnT).$$

The tapped delay line equalizer offers independent control of the frequency characteristic only over a radian frequency range of $\pi/T$ radians/sec, that is, 1/2T Hz. If the chosen T fixes the range much smaller than the channel bandwidth, conflicting requirements on the equalizer result due to the different raw channel characteristics in the frequency increments separated by 1/2T Hz, and only compromise results are obtained. Conversely, if T is picked so that the range greatly exceeds the channel bandwidth it is conceivable that the equalizer will not be efficiently utilized, considering the number of tap points available, by the following reasoning. The desired equalizer characteristic can be considered as the complement of the raw channel characteristic. A periodic distortion with L/2 cycles in the bandwidth 1/2T requires correction by at least a (2L + 1)-tap equalizer, based upon paired echo theory [31], [45]. Thus for a fixed value of L, the value of 1/2T should be made approximately equal to the desired channel bandwidth so that optimum use is made of the finesse available from the (2L + 1) tap points.

When the channel bandwidth considered is 3500 Hz a logical choice of the delay per section is $T = 1/(2 \cdot 3500) = 0.143$ ms. Figure 11 shows the equalized channel error as a function of the delay per section for two parabolic delay distortion cases. The values are each the local minimum in error for variation of the ideal channel delay over a reasonable interval. For case 2 the amplitude of the error dependence on p has been noted to be quite large for T = 0.16 ms and very small for T = 0.10 ms. In the latter situation the effectiveness of the p-selection routine is reduced.

In several runs for which T = 0.15 ms a sharp dip in the equalizer amplitude response and rapid fluctuations in phase are noted in the vicinity of the 1/2T frequency, 3333 Hz. Such perturbations are undesirable since the differential envelope delay at somewhat lower frequencies is unduly affected. Consequently, a value of T = 0.13 ms is chosen for the 3500 Hz channel to extend the controlled bandwidth to 3850 Hz (110%).

Before beginning the discussion of the system performance for various distortion cases some comments are warranted on the accuracy of the simultaneous equation solution performed by the computer (IBM 360/Model 65). The results shown in

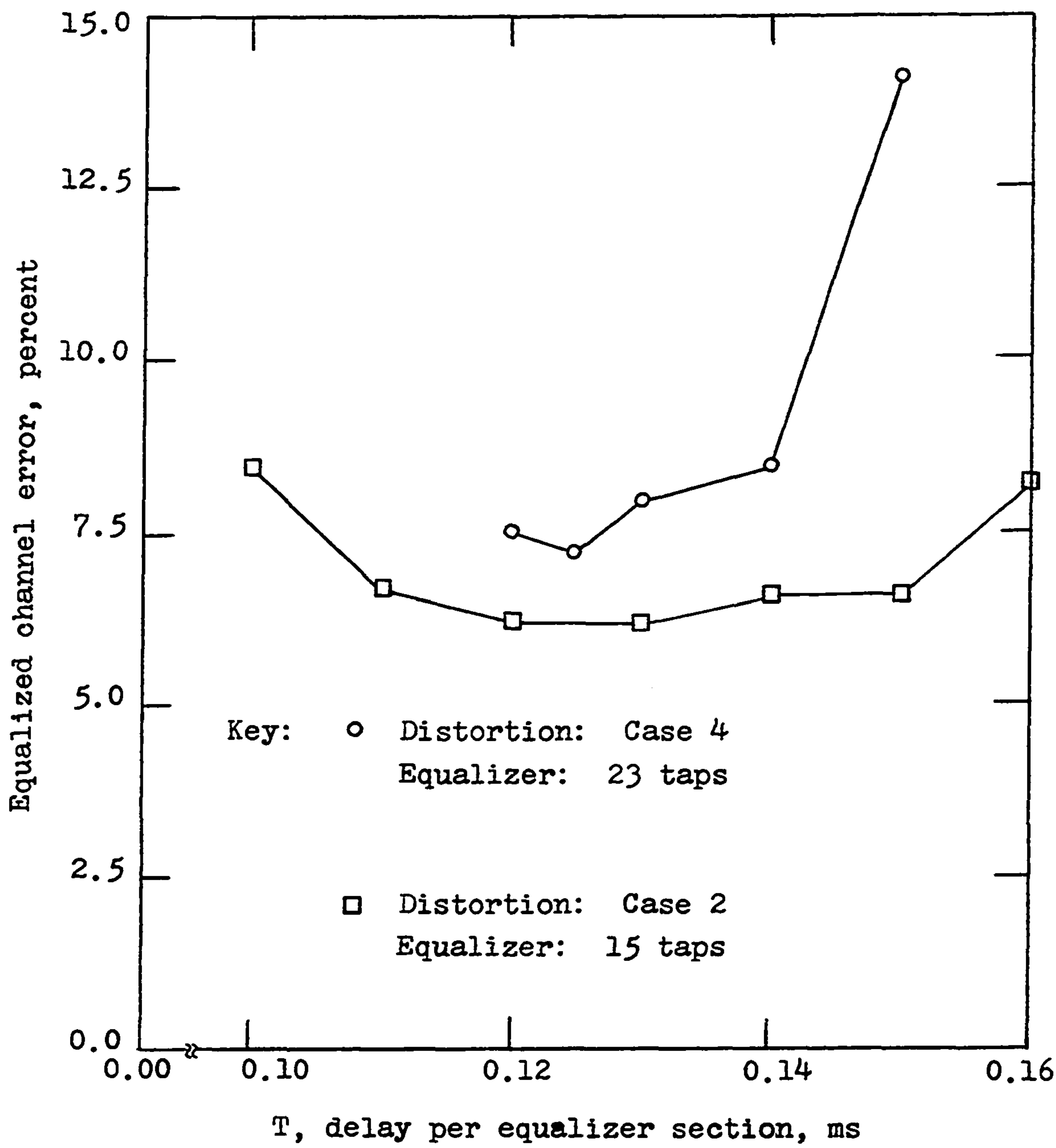


Figure 11. Equalized channel error vs. delay per section, cases 2 and 4

Figure 6 using the point specification algorithm were obtained by the linear equation subroutine SIMQ [21]. This general purpose subroutine uses the Gauss elimination method discussed by Fox [13]. The accuracy of the tap gain solution in the example quoted is such that the equalized channel error calculated at the input data frequency points only is about .001 percent, an extremely low error. It is true that the solution obtained in this case is not a practical one, but this fact arises from the point specification algorithm, not from the simultaneous equation solution process.

The slope specification algorithm runs do not exhibit such small residual errors because the error integration is performed at different frequency locations than those used in the matrix equation derivation. However, checks can be made on the accuracy of tap gain solution by more direct means as described below.

Subroutine SIMQ has been replaced in the equalizer simulation program by subroutine GELS [21] for the system performance runs which are described subsequently. This is also a Gauss elimination method, and it makes use of the coefficient matrix symmetry property discussed in Appendix A. Subroutine GELS requires specification of an internally used tolerance labeled EPS in the program. This tolerance defines how small (relative to the original diagonal elements) the magnitude of the matrix diagonal element used for pivoting at each step in the Gauss elimination may be before the process is terminated without solution. This serves as a precaution against dividing by a nearly-zero number and yielding a solution which is potentially very erroneous. The value EPS = $1 \times 10^{-6}$ is recommended for GELS in the program user's manual for single precision arithmetic. This choice for the investigations of system performance reported here is validated by the following test cases.

The first illustration is provided by the amplitude response correction in the absence of phase distortion. It is readily seen from the equation of the equalizer characteristic that setting the symmetrically located tap gains equal, i.e., $c_{-n} = c_n$ (for $n = 1,2,\ldots,L$) leads to a strictly real response. In this situation the response contribution due to the n-th and -n-th taps is given by

$$
\begin{aligned}
\underline{C}_n(w) &= c_n \cdot \exp(-j\,wnT) + c_n \cdot \exp(+j\,wnT) \\
&= c_n[\cos(-wnT) + j\,\sin(-wnT)] \\
&+ c_n[\cos(+wnT) + j\,\sin(+wnT)] \\
&= c_n[\cos wnT + \cos wnT + j(-\sin wnT + \sin wnT)] \\
&= 2\,c_n \cdot \cos wnT + j \cdot 0.
\end{aligned}
$$

Also $\underline{C}_o(w) = c_o \cdot \exp(-j\,w \cdot 0 \cdot T) = c_o$. The total response is

$$\underline{C}(w) = c_o + \sum_{n=1}^{L} 2\,c_n \cdot \cos wnT,$$

which is real and has zero phase shift at all frequencies.

If the raw channel distortion is purely amplitude distortion with zero differential delay, the proper equalizer characteristic should be purely real, implying that $c_n = c_{-n}$ for $1 \leq n \leq L$. There are twenty-one amplitude distortion only runs examined for distortion cases 5-7, with the number of tap points ranging from 3 to 17. In the computer solution the maximum difference between corresponding tap gains is .0001, this occurring in 14% of the runs. This check is made on the .0001 resolution printout so that in the remaining 86% of the runs the difference shown is zero to four decimal places. Equalizer phase shifts are typically on order of $10^{-8}$ radians. Thus the computer solution accuracy is compatible with the finest resolution conceivable in practical hardware implementation.

As a further test, the amplitude response of a 9-tap equalizer has been computed, assuming the following tap gains: $c_o = 1.0$; $c_{\pm 1} = 0.3$; $c_{\pm 2} = -0.2$; $c_{\pm 3} = 0.0$; $c_{\pm 4} = 0.1$. The 0 to 3200 Hz passband is considered, and $T = 1/(2 \cdot 3200) = 0.156$ ms is chosen as the delay per section. The response consists of periodic distortion components having periods of 6400, 3200, and 1600 Hz, corresponding to the assumed tap gains. The raw channel amplitude response, called case 11, has been set equal to the reciprocal of that for the equalizer; Figure 12 is a plot of this response. The differential envelope delay is specified to be zero at all frequencies.

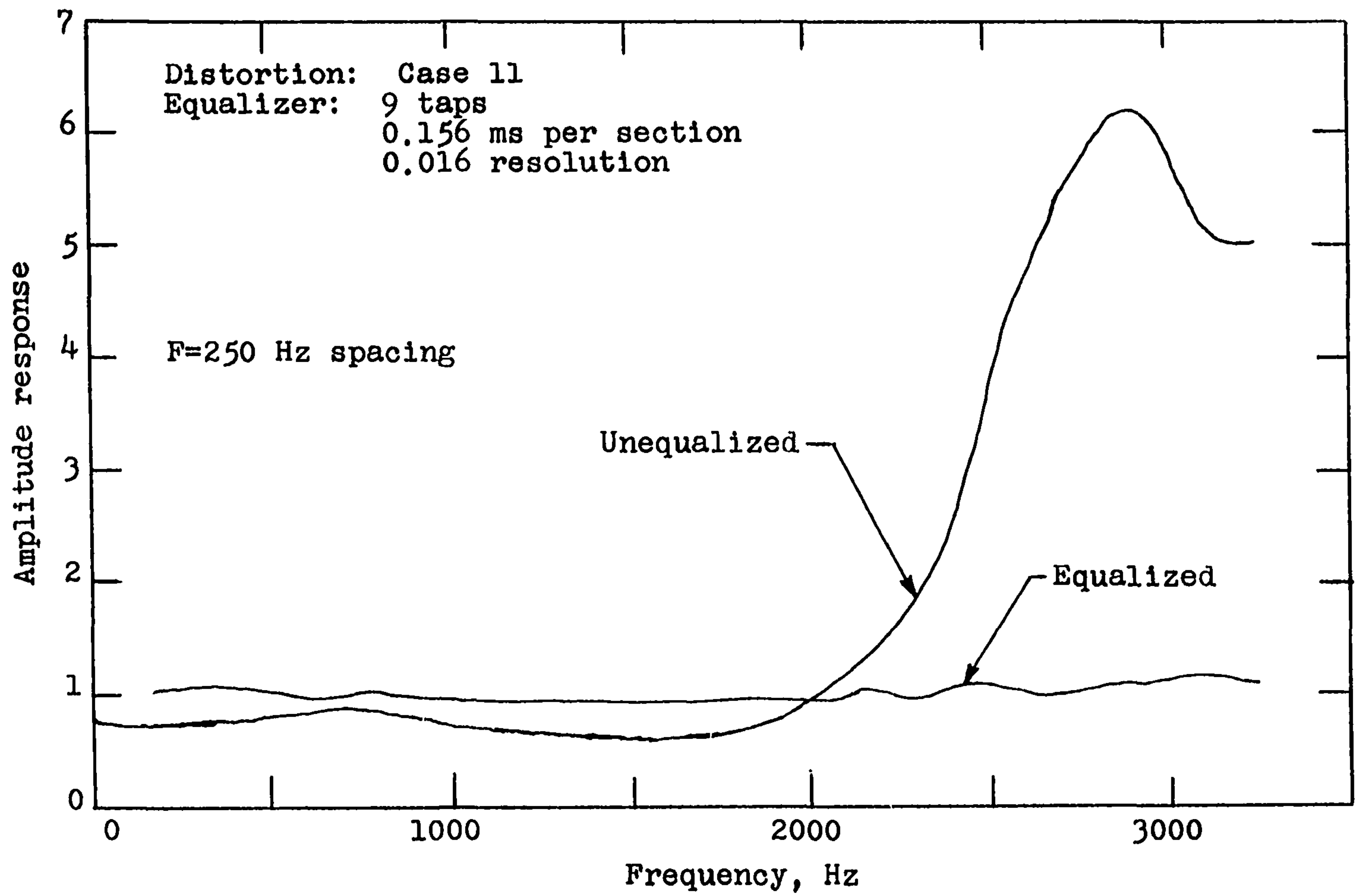


Figure 12. Case 11 amplitude response

(The unequalized channel error is 161 percent.) Table 3 lists the computed tap gains as a function of the measurement data spacing. It is evident that for F=100 Hz the solutions are accurate to two decimal places, which corresponds with the representation of the input data to the program. The equalization error increases with the measurement data spacing. For F=800 Hz there are only two "samples" of the highest-frequency distortion component in the passband, and these two are dependent, so the equalizer channel error of 11.40 percent is a reasonable result.

Table 3. Computed tap gains for distortion case 11

| Spacing F, Hz | Gains $c_{\pm 4}$ | $c_{\pm 3}$ | $c_{\pm 2}$ | $c_{\pm 1}$ | Summing | Result[a] |
|---|---|---|---|---|---|---|
| Ideal | .1000 | .0000 | -.2000 | .3000 | 1.0000 | 0.00 |
| 100 | .0990 | -.0011 | -.1958 | .3032 | .9993 | 1.27 |
| 200 | .0995 | .0011 | -.1914 | .3060 | .9995 | 1.75 |
| 250 | .0963 | -.0012 | -.1862 | .3094 | .9983 | 2.74 |
| 250[b] | .1089 | -.0041 | -.1829 | .3128 | 1.0067 | 2.34 |
| 400 | .0936 | -.0011 | -.1703 | .3189 | .9921 | 4.32 |
| 800 | .0292 | -.0665 | -.1605 | .3129 | .9092 | 11.40 |

[a]Equalized channel rms error in percent, resolution = 0.0001.

[b]Operational mode, N=17 taps. The gains for taps numbered $\pm 5$, $\pm 6$, $\pm 7$, $\pm 8$ are all less than .01 in magnitude and are not shown.

This situation provides an opportunity to demonstrate a special feature of the simulation program. After the tap gains have been quantized using the specified resolution, the outermost taps are examined to determine whether the values are nonzero. Zero-gain outside taps are not required, so that in practice the implementation control circuitry need not connect these when "constructing" the equalizer, thus reducing the total quantity of delay line sections drawn from the array. In the 250 Hz spacing operational mode run noted in Table 3, the estimated number of required tap points is computed to be 17, based upon the 161 percent unequalized channel error. After quantizing, using 0.016 resolution, the program notes that the 8 (total) outside taps are unrequired, so that, in fact, only 9 tap points are needed.

It can be similarily shown that if <u>minor</u> phase correction is required for the case of no amplitude distortion the symmetrically located tap gains should be opposite in sign, i.e., $c_n = -c_{-n}$, provided zero ideal channel (absolute) delay is desired. Such a condition does not arise in this study due to the method of raw channel phase distortion representation and the associated ideal channel mode. However a check on the matrix equation subroutine and algorithm accuracies can be made using a special form of the program which represents the raw channel as perfect. The ideal channel delay is externally specified as follows: $p = K \cdot T$, where $K = 0,1,\ldots,L$, an integer, and $T$ = delay per section. Then the equalizer is constrained to provide exactly K·T seconds of delay at all frequencies, and the impulse response is to be

$c(t) = \delta(t - KT)$.

Equivalently this requires, by the Fourier transform,

$\underline{C}(w) = \exp(-j\ wKT)$.

The solution for the tap gain vector is therefore

$c_n = 0$ for $n \neq K$

$c_n = 1$ for $n = K$ where again $-L \leq n \leq L$, n an integer.

Several runs have been made in which the value of K ranges up to 4 sections. The program tap gain solutions typically satisfy the above equations to 6 decimal place accuracy, which is certainly adequate for this investigation and practical implementation.

Figure 13 shows the amplitude and phase characteristic of distortion case 12. This has been developed as the complement to 7-tap equalizer having tap gains as follows:

$c_{-3} = 0.1$; $c_{-2} = -0.2$; $c_{-1} = 0.3$; $c_o = 1.0$; $c_1 = -0.3$;

$c_2 = 0.2$; $c_3 = -0.1$.

Notice the antisymmetry of the tap gains suggests primarily phase distortion correction ability. Again the distortion has periodic components, in this case with periods of 6400, 3200, and 2133 Hz. Table 4 lists the computed tap gains for several values of F. Two decimal-place accuracy is obtained here also.

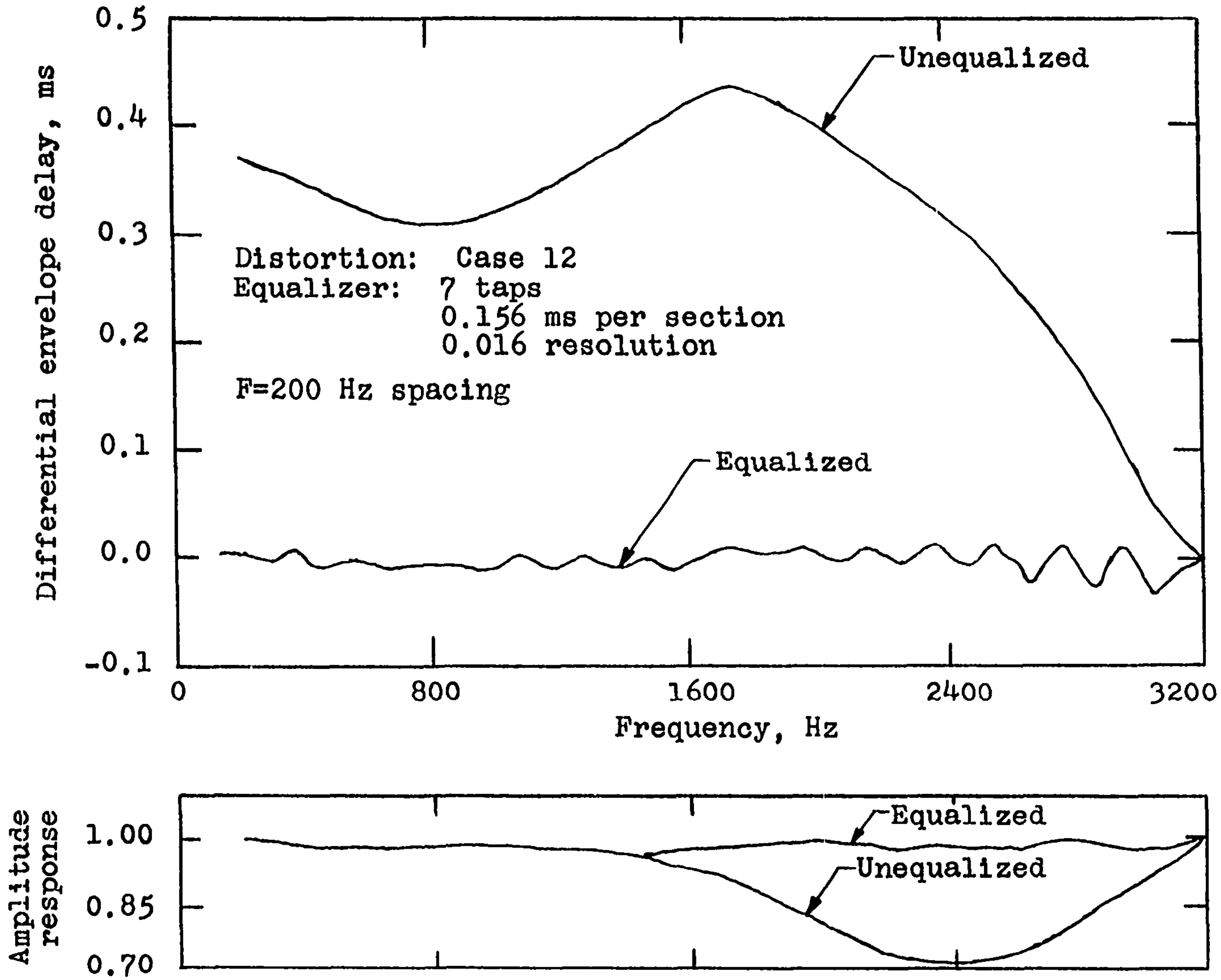


Figure 13. Case 12 characteristic

Table 4. Computed tap gains for distortion case 12

| F, Hz | $c_{-3}$ | $c_{-2}$ | $c_{-1}$ | Gains Summing | $c_1$ | $c_2$ | $c_3$ | Result[a] |
|---|---|---|---|---|---|---|---|---|
| Ideal | .1000 | -.2000 | .3000 | 1.0000 | -.3000 | .2000 | -.1000 | 0.00 |
| 100 | .1042 | -.1957 | .3032 | 1.0109 | -.3031 | .2049 | -.1038 | 0.91 |
| 200 | .1006 | -.1844 | .2828 | 1.0152 | -.2923 | .2015 | -.0966 | 1.01 |
| 400 | .0939 | -.1864 | .3160 | 1.0085 | -.2954 | .2214 | -.0974 | 2.71 |
| 800 | .0626 | -.1870 | .3089 | 1.0312 | -.2515 | .2433 | -.0833 | 6.61 |

[a]Equalized channel rms error in percent, resolution = 0.0001.

The paired-echo theory application to equalizers was mentioned previously. In case 11 the 1600 Hz period distortion component has two cycles in the passband. For this case one sets $L/2 = 2$ cycles, giving $L = 4$ and $N = 9$ as the required number of taps. The low equalized channel error achievable using 9 taps confirms this. Similarily, in distortion case 12 the 2133 Hz component has $1\frac{1}{2}$ cycles, so that $L = 2 \cdot 1\frac{1}{2} = 3$ and $N = 7$, as verified. The uniform weighting is used in these special cases to allow equal emphasis on errors throughout the passband.

In distortion cases 1-7 (refer to Table 1) the raw channel distortion is modeled as either parabolic delay or linear dB attenuation, according to the format shown in Figure 5. All runs utilize 250 Hz spacing of the frequency-domain data points and error weighting function number 1 from Figure 4.

The results for delay distortion equalization are plotted in Figure 14 as equalized channel error vs. the maximum raw channel delay. Emphasis has been placed on using a large number of equalizer taps in an attempt to achieve C4 specification compliance. As mentioned previously the delay per section is 0.130 ms in these runs. The slope specification algorithm and the (local-minimum error vs. p) search are employed. For small N the amplitude response is distorted by the equalizer while reducing delay distortion to obtain the minimum error. Table 5 lists the equalization results for case 1. Compliance with C4 grade specifications is indicated.

Table 5. Equalization results[a] for distortion case 1

| N, tap points | Equalizer[b] resolution: .0001 | .0160 |
|---|---|---|
| None | (63.0 unequalized) | |
| 3 | 26.0 | 26.0 |
| 5 | 14.5[c] | 14.6[c] |
| 7 | 11.2[c] | 11.3[c] |
| 9 | 3.7[c] | 4.0[c] |
| 11 | 3.2[c] | 3.7[c] |
| 13 | 1.8[c] | 2.3[c] |
| 15 | 1.6[c] | 2.3[c] |

[a]Equalized channel rms error in percent.

[b]Equalizer delay per section: T = 0.13 ms.

[c]Corresponding characteristic complies with C4 grade specifications.

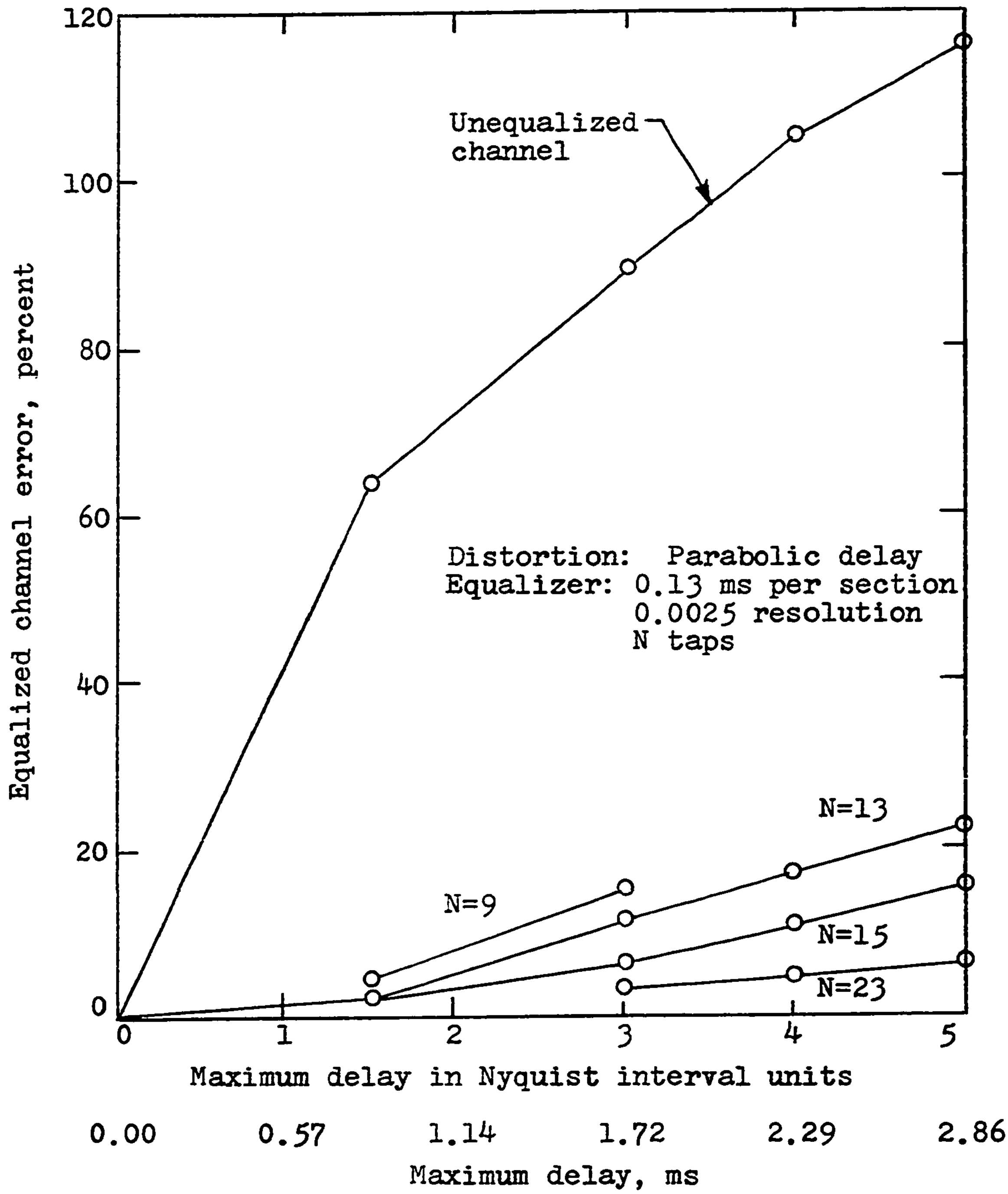


Figure 14. Equalized channel error vs. maximum of parabolic delay, cases 1-4

Table 6. Equalization results[a] for distortion cases 2-4

| N, tap points | Equalizer resolution: .0001 | .0025 | .0250 | .1000 | Delay sub-runs[b] Mean | Std. dev. |
|---|---|---|---|---|---|---|
| | | | Case 2 | | | |
| None | (88.4 unequalized) | | | | | |
| 9 | 15.9 | 15.9 | 16.2 | 24.0 | 17.7 | 1.2 |
| 11 | 13.0 | 13.1 | 13.5 | 16.9 | 14.2 | 0.5 |
| 13 | 11.1[c] | 11.1[c] | 11.2[c] | 14.7[c] | 13.5[c] | 1.9 |
| 15 | 6.1[c] | 6.1[c] | 7.2[c] | 11.4[c] | 9.8[c] | 2.2 |
| 17 | 6.0[c] | 6.1[c] | 6.4[c] | 14.1[c] | 12.5 | 1.7 |
| 19 | 4.0[c] | 4.0[c] | 4.7[c] | 16.8[c] | 9.8[c] | 4.5 |
| 21 | 3.9[c] | 3.9[c] | 6.1[c] | 17.4[c] | 9.0[c] | 2.6 |
| 23 | 3.4[c] | 3.4[c] | 4.2[c] | 11.6[c] | 11.3[c] | 1.5 |
| | | | Case 3 | | | |
| None | (105.0 unequalized) | | | | | |
| 13 | 16.8 | 16.8 | 17.1 | 22.4 | 19.1 | 1.0 |
| 15 | 10.4 | 10.4 | 10.6 | 19.2 | 13.2 | 1.8 |
| 17 | 10.2 | 10.2 | 10.8 | 18.2 | 12.9 | 1.2 |
| 19 | 6.3[c] | 6.4[c] | 7.7[c] | 12.8 | 10.9 | 1.5 |
| 21 | 6.0[c] | 6.0[c] | 6.2[c] | 15.8 | 11.4 | 1.8 |
| 23 | 4.5[c] | 4.5[c] | 5.8[c] | 14.1 | 10.3 | 3.7 |
| 25 | 4.4[c] | 4.4[c] | 6.4[c] | 16.0 | 11.1[c] | 2.7 |

[a]Equalized channel rms error in percent.

[b]Equalizer delay per section variation: mean = 0.13 ms, standard deviation = 6%, rejection = 9%, sample size = 5 sub-runs, resolution = 0.0025.

[c]Corresponding characteristic complies with C4 grade specifications.

Table 6. (Continued)

| N, tap points | Equalizer resolution: .0001 | .0025 | .0250 | .1000 | Delay sub-runs[b] Mean | Std. dev. |
|---|---|---|---|---|---|---|
| | | | Case 4 | | | |
| None | (116.0 unequalized) | | | | | |
| 15 | 15.2 | 15.2 | 15.2 | 22.4 | 18.8 | 0.4 |
| 17 | 13.5 | 13.5 | 14.0 | 20.1 | 17.0 | 1.4 |
| 19 | 10.0 | 10.0 | 10.3 | 18.4 | 17.9 | 3.4 |
| 21 | 8.8 | 8.8 | 10.0 | 21.6 | 13.5 | 3.1 |
| 23 | 6.1[c] | 6.1[c] | 7.0[c] | 20.2 | 12.7 | 3.5 |
| 25 | 6.0[c] | 6.0[c] | 6.5[c] | 14.5 | 9.4 | 1.4 |
| 27 | 4.9[c] | 4.9[c] | 6.0[c] | 16.1 | 11.0[c] | 2.3 |
| 29 | 4.9[c] | 4.9[c] | 6.1[c] | 16.6 | 11.5 | 2.3 |
| 31 | 5.0[c] | 5.0[c] | 5.8[c] | 16.2[c] | 12.6 | 2.8 |

A resolution value of 0.0025 has been chosen for some equalizers reported by Lucky [27], [28]. As shown in Table 6 for distortion cases 2-4 the results for 0.0001 and 0.0025 resolution are essentially identical. The effect of coarser resolution can be seen in the table; these results have been obtained by the program setting the resolution equal to 0.0250 and 0.1000 in turn. The 0.0250 resolution performance is close to that for 0.0025 resolution in both residual error and C4 specification compliance, but the same is not true for 0.1000 resolution. In the latter situation the error "asymptote" for large N is about 10% above the former.

Table 6 also displays the effect of variations in the delay per equalizer section for 0.0025 resolution. A delay distribution standard deviation of 6% and a rejection tolerance of 9% are used in this investigation. The mean of the distribution is set at unity, that is, 0.13 ms. According to this model 16% of the normal distribution values exceed the tolerance and are replaced by new values, since the rejection tolerance is $1\frac{1}{2}$ times the standard deviation. Five sub-runs are made in each situation, picking random delay values each time. The mean and standard deviation of the equalized channel error are shown. The specification compliance symbol is used when four or more of the Monte Carlo technique sub-runs comply with C4 specifications. In every sub-run the perturbated equalizer system error exceeds that of the perfect equalizer. Additional discussion of these results is given subsequently.

The same parametric studies of system performance for amplitude distortion are represented in Figure 15 and Table 7 with similar comments applying.

The number of tap points required to achieve the same reduced error is greater for the frequency-domain general-purpose algorithm than for the time-domain synchronous-data method evaluated by Lucky _et_ _al_. [31]. This comment applies to both the parabolic delay and the linear (dB) attenuation distortions. One reason is that the error here is evaluated at frequencies in addition to those used for the input measurements. Also, it is known that general-purpose

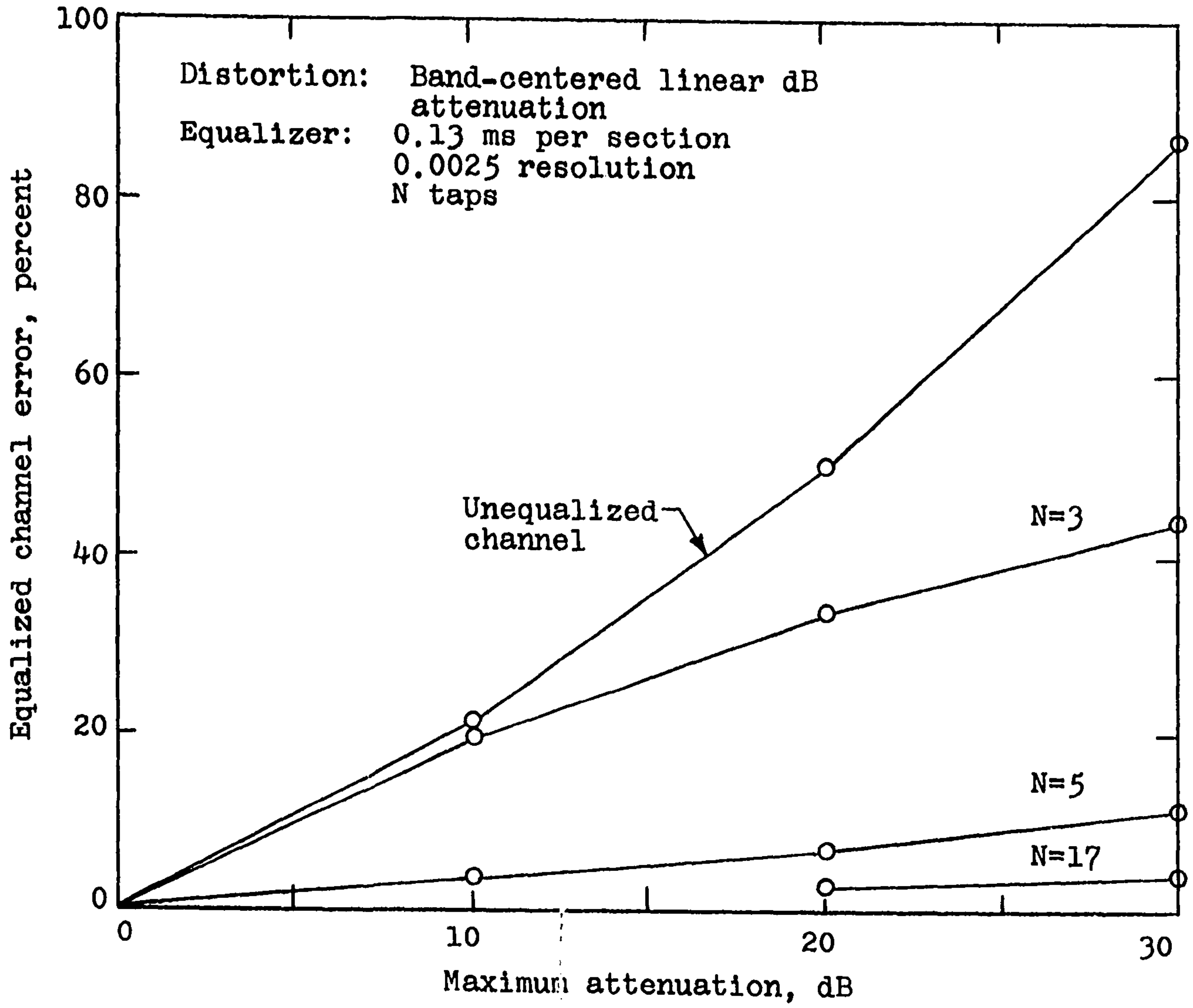


Figure 15. Equalized channel error vs. maximum attenuation, cases 5-7

Table 7. Equalization results[a] for distortion cases 5-7

| N, tap points | Equalizer resolution: .0001 | .0025 | .0250 | .1000 | Delay sub-runs[b] Mean | Std. dev. |
|---|---|---|---|---|---|---|
| | | | Case 5 | | | |
| None | (21.6 unequalized) | | | | | |
| 3 | 19.1 | 19.1 | 19.2 | 20.0 | 19.3 | 0.02 |
| 5 | 3.7[c] | 3.7[c] | 4.8[c] | 9.0[c] | 4.5[c] | 0.7 |
| 7 | 3.2[c] | 3.2[c] | 3.5[c] | 8.1[c] | 4.1[c] | 0.5 |
| 9 | 3.2[c] | 3.2[c] | 5.2[c] | 8.1[c] | 4.1[c] | 0.3 |
| 11 | 3.1[c] | 3.2[c] | 5.0[c] | 9.0[c] | 4.3[c] | 0.2 |
| | | | Case 6 | | | |
| None | (49.2 unequalized) | | | | | |
| 3 | 33.4 | 33.4 | 33.6 | 33.8 | 33.3 | 0.1 |
| 5 | 6.4[c] | 6.4[c] | 7.0[c] | 14.1 | 9.1[c] | 1.6 |
| 7 | 6.0[c] | 6.0[c] | 7.6[c] | 21.0 | 8.6[c] | 2.1 |
| 9 | 4.9[c] | 4.9[c] | 6.3[c] | 18.3 | 8.7[c] | 2.2 |
| 11 | 4.9[c] | 4.9[c] | 6.3[c] | 18.3 | 9.5[c] | 2.2 |
| 13 | 3.2[c] | 3.2[c] | 6.6[d] | 14.2 | 9.8[c] | 2.6 |
| 15 | 2.8[c] | 2.9[c] | 5.3[c] | 13.6 | 9.2[c] | 3.1 |
| 17 | 2.8[c] | 2.9[c] | 6.1[d] | 14.2 | 7.8[c] | 2.8 |

[a]Equalized channel rms error in percent.

[b]Equalizer delay per section variation: mean = 0.13 ms, standard deviation = 6%, rejection = 9%, sample size = 5 sub-runs, resolution = 0.0025.

[c]Corresponding characteristic complies with C4 grade specifications.

[d]Non-compliance caused by 1750 Hz amplitude response only.

Table 7. (Continued)

| N, tap points | Equalizer resolution: .0001 | .0025 | .0250 | .1000 | Delay sub-runs[b] Mean | Std. dev. |
|---|---|---|---|---|---|---|
| | | | Case 7 | | | |
| None | (86.2 unequalized) | | | | | |
| 5 | 11.7 | 11.7 | 14.0 | 18.0 | 16.2 | 1.6 |
| 7 | 11.5 | 11.6 | 14.6 | 18.0 | 16.8 | 2.3 |
| 9 | 7.3[c] | 7.4[c] | 7.7[c] | 89.6 | 11.1 | 2.2 |
| 11 | 7.3[c] | 7.4[c] | 7.7[c] | 89.6 | 21.0 | 4.4 |
| 13 | 4.0[c] | 4.0[c] | 10.2[c] | 22.1 | 27.2 | 15.2 |
| 15 | 3.4[c] | 3.6[c] | 8.9[d] | 24.6 | 13.5 | 4.9 |
| 17 | 3.3[c] | 3.3[c] | 7.2[c] | 22.1 | 22.9 | 4.7 |

equalization is not as efficient as specific known-sample-interval equalization, although the former may be economically justified.

Both case 7 (amplitude distortion) and case 2 (delay distortion) indicate a raw channel error of 85%. Note that satisfactory equalization (C4 compliance) is possible with 9 taps in the amplitude distortion case whereas 13 taps are required in the delay distortion case. Amplitude distortion is easier to correct because no phase distortion is introduced by the perfect equalizer, for reasonable resolution values. On the contrary, major phase distortion correction results in significant amplitude response deviations even for the optimum solution, hence a larger number of taps is required to offset this effect. The staircase function previously mentioned for picking the number of tap points is based upon these amplitude distortion cases. It has been chosen to give an optimistic estimate, so that the minimum number is likely to be found.

It is to be noted from Table 7 that in a few situations use of 0.0250 resolution results in unexpected noncompliance with specifications (case 6, N = 13 and 15 and case 7, N = 15). The location of the noncompliance is the frequency of the raw channel amplitude response peak--this is not considered a significant failure. The high error obtained for 9 and 11 tap points using 0.1000 resolution in case 7 (Table 7) arises from a $180^{\circ}$ phase change at mid-band in the equalizer response--this appears to be an exceptional situation brought about by unrealistic tap gain resolution.

The following observation is made from Table 7: given a fixed number of sections, N, the equalizer delay variation is generally more disruptive (leads to larger error mean and standard deviation) for more severe raw channel distortion conditioning. This is because the "large n" tap gains must be of greater magnitude to accomplish the task. It is apparent from the random walk analogy that the variance of the total delay from the equalizer midpoint to tap n increases with n, because this delay is the sum of n independent "normally-distributed" delay values in the model used. The consequence is degradation of the equalization when these tap point gains are assigned significant values.

Equalizer delay variations introduce phase distortion in the amplitude correction cases because the symmetrically located tap points do not possess exactly the same delay relative to the midpoint. On the other hand, relaxing the tap gain resolution in the amplitude-correcting perfect equalizer usually does not result in phase distortion because symmetrically located tap gains are quantized identically.

Based upon Tables 6 and 7 it is concluded that reduction of the channel error to below 10% is compatible with achieving C4 grade specification compliance. This is substantiated by a study of the equalizer delay variation results: in only five percent of the Monte Carlo runs where C4 specifications are not met is the channel error lower than 10%. Thus the equalizer algorithm for minimizing the channel error also leads to compliance with frequency-domain amplitude and delay standards. Figure 16 depicts the equalized channel characteristic for distortion case 8 (combined amplitude and delay distortion) and illustrates the equalizer algorithm utility in conditioning channels for general purpose transmission usage.

Weighting number 1 (from Figure 4) is employed in the parametric studies just reported. It is designed to give equal emphasis throughout the 1000 to 2600 Hz range where the delay specification is the tightest. Weighting number 2 is a raised-cosine function--one corresponding to the

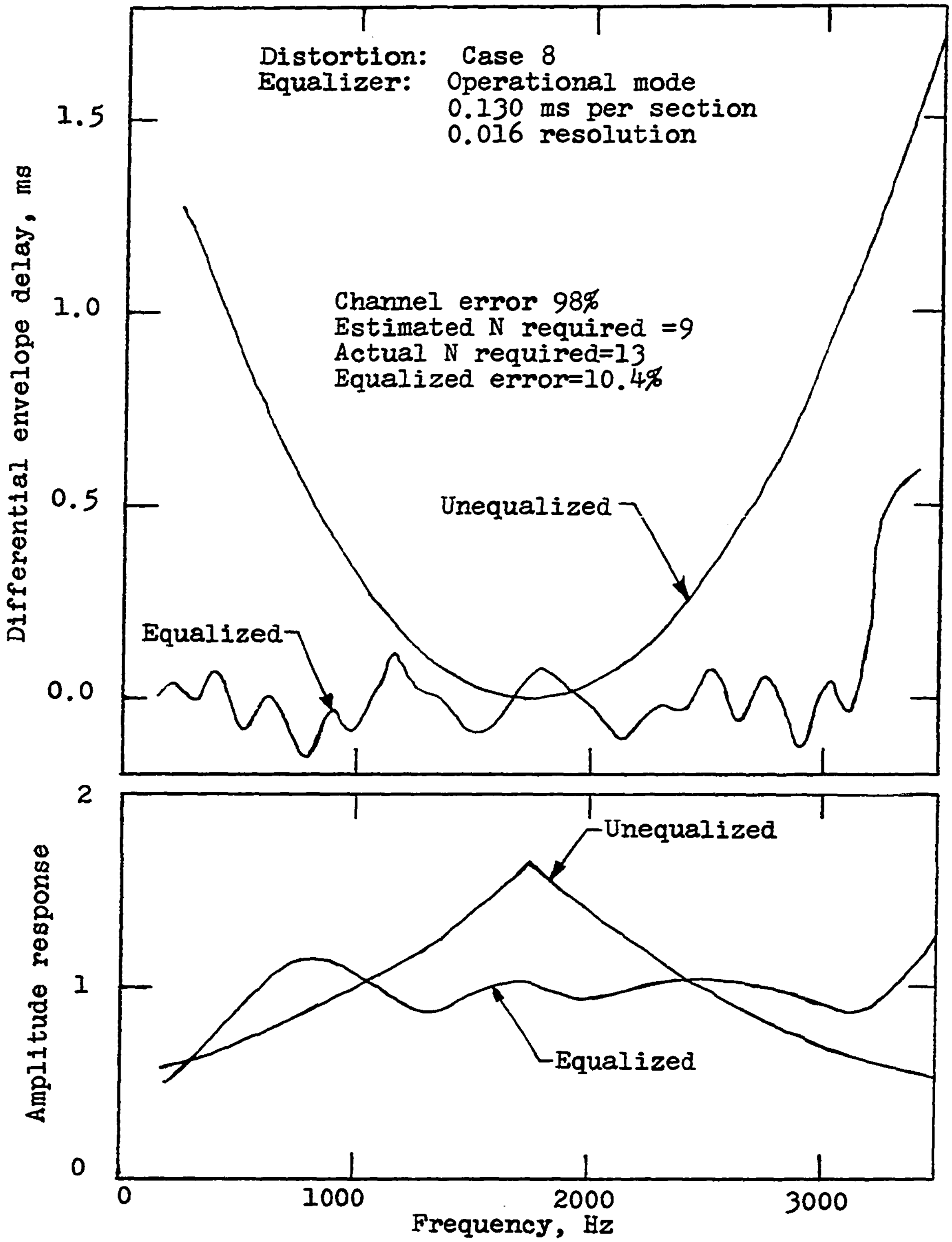


Figure 16. Case 8 characteristics

optimum pulse spectrum under certain Nyquist criteria for double-sideband amplitude modulation [5]. Table 8 compares

Table 8. Effect of weighting on equalization results[a], cases 3 and 7

| Tap points | Weighting #1 | Weighting #2 |
|---|---|---|
| | Case 3 | |
| none[b] | 105.0 | 107.4 |
| 13 | 16.8 | 9.9 |
| 15 | 10.4 | 6.7[c] |
| 17 | 10.2 | 4.8[c] |
| 19 | 6.2[c] | 3.8[c] |
| | Case 7 | |
| none[b] | 86.2 | 105.0 |
| 5 | 11.6 | 7.4 |
| 7 | 11.5 | 7.2 |
| 9 | 7.3[c] | 5.6[c] |
| 11 | 7.3[c] | 5.6[c] |

[a]Equalized channel rms error in percent, resolution = 0.0001.

[b]Unequalized channel.

[c]Corresponding characteristic complies with C4 grade specifications.

the equalization results for distortion cases 3 and 7 using two weightings. The differences are not very significant, but there is a tendency for weighting number 2 to cause accomplishment of the equalization task with fewer taps.

As previously mentioned all results reported for distortion cases 1-7 are for 250 Hz spacing of the frequency-domain data. Only when the point specification algorithm is used is the 100 Hz spacing found to be significantly better for equalization solution. For instance, with 100 Hz spacing the erratic performance displayed in Figure 6 is eliminated. However, the slope algorithm is the preferred one and the 250 Hz interval is adequate. To substantiate this, Table 9 lists comparitive data on equalized channel error in distortion cases 4 and 7 for the two spacings--there is essentially no difference in the equalizer performance.

For some applications the 250 Hz spacing of data may not adequately characterize the channel because of severe variations in the delay or amplitude response at intermediate frequencies. In such cases the distortion spectral bandwidth is greater than considered here and a longer delay line is required for suitable equalization. For example, suppose the shortest distortion-component period is 100 Hz. Certainly measurement data at 250 Hz increments will not represent this component with fidelity. However, this component requires a 141-tap equalizer for complete equalization. The equalizer algorithm will equalize the channel only as represented by the insufficient number of frequency-domain points.

Table 9. Effect of measurement data spacing on equalization results[a], cases 4 and 7

| Tap points | Spacing, F: 100 Hz | 250 Hz |
|---|---|---|
| | Case 4 | |
| None[b] | 110.0 | 116.0 |
| 15 | 15.8 | 15.2 |
| 17 | 13.5 | 13.5 |
| 19 | 11.4 | 10.0 |
| 21 | 10.1[c] | 8.8 |
| | Case 7 | |
| None[b] | 82.5 | 86.2 |
| 5 | 11.9 | 11.7 |
| 7 | 11.7 | 11.5 |
| 9 | 7.1[c] | 7.3[c] |
| 11 | 7.1[c] | 7.3[c] |
| 13 | 3.5[c] | 4.0[c] |

[a]Equalized channel rms error in percent, resolution = 0.0001.

[b]Unequalized channel.

[c]Corresponding characteristic complies with C4 grade specifications.

As seen from Tables 4 and 5, previously discussed, the algorithm performs well given a reasonable representation.

The 0.0250 resolution appears to have potential merit based on the above findings. For implementation one might select the tap gain range as -1.0 to +1.0 and utilize a

7 digit control word. One digit is needed for the sign so that there remain $(2^6 - 1) = 63$ non-zero quantizing levels. Hence the resolution can be $1/63 = 0.016$, a reasonable compromise between 0.0025 and 0.0250 resolution. Previous studies reported in the literature [28] have considered a tapering of the tap gain resolution, assigning the finest resolution to the tap gains far removed from the midpoint since those gains take on the smallest range of values. This technique is especially used when the channel is conditioned for known-baud synchronous data transmission, and it is desired to completely eliminate intersymbol interference at the sampling instants with lesser emphasis on the impulse response for intermediate time values. This investigation does not suggest such a technique since the system performance improvement in changing from 0.0250 to 0.0001 resolution is not very significant, due to the method of error evaluation. Furthermore, tapering is not feasible for a system which constructs an equalizer from a mutually accessible array of delay sections with associated tap gain controls, since a particular section could be the midpoint on one day and the far removed tap on a subsequent day.

In this investigation the required summing amplifier gain ranges from 0.4 to 2.6. Utilizing an 8 digit control word allows a resolution of 0.016 (as for the tap gains) and a gain range of 0 to 4.0 since only positive values are needed. Conceivably an auxiliary amplifier would be employed for correcting the raw channel gain or loss to the level required at the installation.

Table 10 lists some additional results obtained in the

Table 10. Additional equalization results[a] for distortion case 4

| Tap points | Invest. mode Res.: .0001 | Operational mode Resolution: .0001 | Operational mode Resolution: .0160 | Delay sub-runs[b] Mean | Delay sub-runs[b] std. dev. |
|---|---|---|---|---|---|
| 15 | 15.2 | 15.9 | 16.1 | 16.3 | 0.2 |
| 17 | 13.5 | 12.8 | 13.1 | 13.8 | 0.5 |
| 19 | 10.0 | 10.2 | 10.4 | 11.2 | 0.4 |
| 21 | 8.8 | 10.2 | 10.4 | 11.2 | 0.4 |
| 23 | 6.1[c] | 6.2 | 6.4 | 6.9 | 0.4 |
| 25 | 6.0[c] | 6.0 | 6.4 | 7.5 | 0.4 |
| 27 | 4.9[c] | 4.4[c] | 4.8[c] | 5.2[c] | 0.3 |

[a]Equalized channel rms error in percent.

[b]Equalizer delay per section variation: mean = 0.13 ms, standard deviation = 2%, rejection = 3%, sample size = 5 sub-runs, resolution = 0.0160.

[c]Corresponding characteristic complies with C4 grade specifications.

operational mode for case 4. It shows that this mode produces essentially the same equalized channel error as does the investigation mode, the latter employing the search for a local minimum error as a function of p. Such is not likely the situation when the number of tap points is not adequate for satisfactory equalization; however, in practice the operational mode is to be preferred since it accomplishes the same task in less computations. It is also evident in the table that the 0.0160 resolution is justified in this case. The equalizer delay distribution standard deviation and rejection tolerance used are 2 percent and 3 percent respectively. This appears to yield satisfactory performance by the simulation model.

Figure 17 shows the characteristics of distortion case 13 before and after equalization using 35 taps. Results for other values of N are shown in Table 11. Based upon the

Table 11. Equalization results[a] for distortion case 13

| Tap points | Equalizer resolution: .0001 | .0160 | Delay sub-runs[b] Mean | Std. dev. |
|---|---|---|---|---|
| None | (114.4 unequalized) | | | |
| 11 | 59.8 | 59.8 | 60.9 | 0.9 |
| 13 | 53.8 | 54.0 | 56.7 | 1.7 |
| 15 | 45.4 | 45.6 | 46.7 | 0.7 |
| 17 | 37.1 | 37.4 | 43.3 | 0.7 |
| 19 | 24.0 | 24.2 | 26.8 | 1.1 |
| 21 | 20.5 | 21.0 | 29.7 | 1.2 |
| 23 | 14.8 | 17.4 | 27.3 | 5.8 |
| 25 | 14.6 | 17.2 | 21.2 | 4.6 |
| 27 | 12.5 | 14.6 | 18.5 | 1.4 |
| 35 | 9.3 | 14.0 | 19.0 | 1.5 |

[a]Equalized channel rms error in percent for error weighting #2.

[b]Equalizer delay per section variation: mean = 0.13 ms, standard deviation = 2%, reject = 3%, sample size = 5 subruns, resolution = 0.0160.

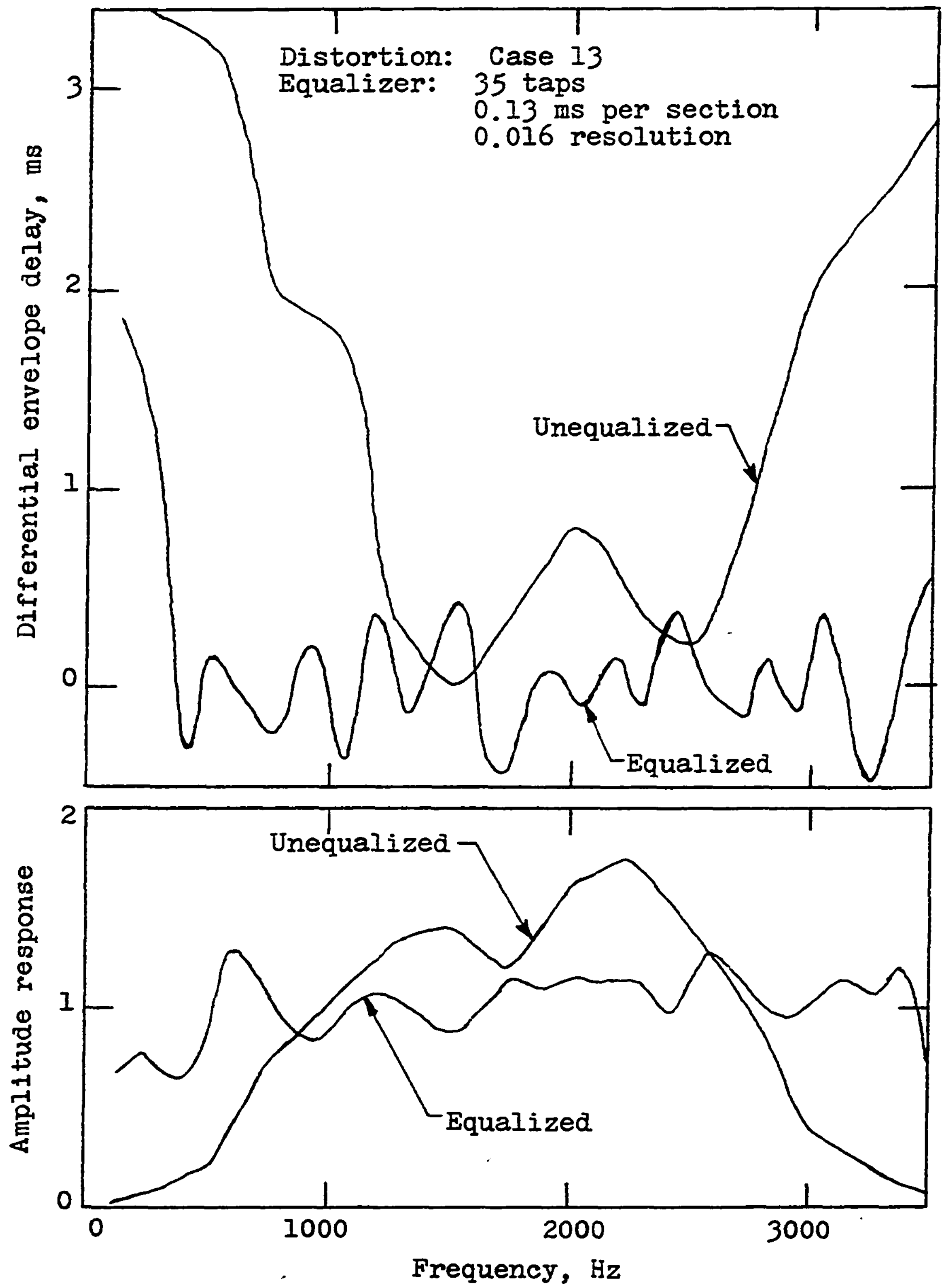


Figure 17. Case 13 characteristics

raw channel error of 114% it is estimated that 11 taps are required to achieve specification compliance, although the number actually needed is somewhat in excess of 35. It is apparent that the error is a monotonically decreasing function of N but the distortion has many components present. For large N, the effect of the 0.0160 resolution contrasted to 0.0001 is significant, suggesting a somewhat smaller value is warranted. The effect of the equalizer delay variation is likewise not negligible. This distortion case is somewhat more severe than the typical channel reported by Alexander *et al*. [1].

No attempt has been made to model the channel characteristic measurement errors in this study. Since the equalization is derived from one discrete (although potentially erroneous) set of measurements there is no possibility of the equalizer algorithm iterating indefinitely in response to noise, as is the case with on-line time-domain equalizers. Measurement errors in the amplitude response cause a bias in the equalizer error in the neighborhood of the error frequency. Differential delay measurement errors give rise to a phase slope error at high frequencies. Although the slope error should disappear over much of the bandwidth when the derivative is taken to obtain the differential delay, it is conceivable that the resultant equalization error caused is not removable. This suggests an alternate scheme which uses the mid-frequency delay data (inherently the most accurate for the commercial test equipment in mind) as the starting point for the phase simulation to circumvent the above stated problem.

# V. SUMMARY AND CONCLUSIONS

The feasibility of utilizing a limited number of frequency-domain channel characteristic measurements in computing the optimum settings for general-purpose equalization using a tapped delay line is demonstrated here. The computation algorithm is derived from minimizing the equalized channel error as a function of the tap gain settings. This leads to a system of simultaneous linear equations which is expressed in matrix equation form. The ideal channel delay, p, is not treated as an unknown in this method, to avoid encountering nonlinear equations. Rather, a first-order approximation to the channel phase function yields a good estimate for an initial choice for p to be used in computing the matrix equation elements. In the investigation mode the process is extended by iteratively decreasing the value of p and computing the equalized channel error until a local-minimum error is found. Because there is little variation of the error with p for well-equalized channels this process is not needed in operational utilization of the algorithm.

The approach used in the research is to simulate a model of the tapped delay line equalizer using a digital computer program labeled EQLB. This approach allows verification of the algorithm and determination of the effects of various equalizer parameters upon the system performance. Several special distortion cases are considered--these all verify the algorithm validity and the system of equations solution accuracy.

It is demonstrated for several typical channel distortion cases that as the number of tap points is increased the equalized channel error decreases monotonically. All channel error values are scaled, using a realistic reference error normalization. For most situations, reduction of the error to below 10 percent is accompanied by the characteristic complying with C4 grade circuit specifications. This confirms the ability of the method to equalize channels for satisfactory general-purpose use.

Both parabolic envelope delay and linear (dB) attenuation distortions, centered in the passband, are considered as sample distortion cases. The unequalized channel distortion errors are found to be comparable to those reported by Lucky et al. [31] when time-domain mean-square distortion is evaluated at synchronous data transmission sampling instants. The number of tap points required to achieve the same reduction in error is greater for the frequency-domain general-purpose algorithm than for Lucky's time domain synchronous-data method. The error here is evaluated at frequencies in addition to the specified measurement frequencies.

The equalization is somewhat dependent upon T, the nominal delay per section. A value equal to or slightly less than 1/(2·BW), where BW is the low-pass bandwidth to be equalized, is used in this investigation. The study verifies the relationship between the number of distortion-component cycles in the $1/(2 \cdot T)$ bandwidth and the required number of tap points for complete equalization, based upon paired-echo theory, for certain selected distortions.

For a moderate number of delay sections a suitable resolution is found to be 0.016, which can be implemented by a 7-digit control word for the tap gains and an 8-digit word for the summing amplifier. However, if the design application is such that 30 or more tap points are needed the equalizer resolution should be improved to perhaps 0.004, requiring two additional digits per control word. Otherwise the equalization error may be asymptotic to 10 or 15 percent when many tap points are needed.

Similarily, a 2 percent standard deviation in the equalizer delay per section, with the manufacturing rejection tolerance set a 3 percent, yields near optimum performance for a moderate number of taps. This assumes that the mean of the distribution represents the desired nominal delay. Again, if the number of delay sections in the design is more than 30 the rejection tolerance should be reduced in order to prevent significant degradation of performance due to this factor.

It is apparent that the 250 Hz spacing of frequency-domain measurement data is satisfactory for most of the channel distortions examined, when using the slope-specification algorithm. There can exist peaks or nulls in the channel characteristic which are not adequately represented by this spacing. In this situation the algorithm still is applicable but the equalization may not be suitable for general-purpose transmission. If the shortest distortion-component period is 250 Hz there is only one measurement sample per frequency period, a very marginal condition. This distortion component requires $4 \cdot 14 + 1 = 57$ tap points for complete equalization, assuming 3500 Hz bandwidth and the corresponding $1/(2 \cdot BW)$ delay per section of the device. More realistically, if two measurements per period are required for adequate representation the shortest distortion-component period is 500 Hz, leading to 29 tap points for complete equalization under the assumptions above. This is likely the reason the results for distortion case 13 improve very gradually with increasing N for $N > 25$ -- the 250 Hz spacing of measurement data is too wide in this case.

As in other engineering designs, the goals must be established and considered when making the trades between performance capability and cost. In the general-purpose equalizer model studied here it is evident that the worst-case distortions to be equalized must be defined. Then proper choices for the design parameters such as resolution, delay-line accuracy, and frequency-domain measurement spacing can be made, based, at least to a degree, upon the results of this study.

## VI. LITERATURE CITED

(1) A. A. Alexander, R. M. Gryb and D. W. Nast, "Capabilities of the telephone network for data transmission," Bell Sys. Tech. J., vol. 39, May 1960, pp. 431-476."

(2) F. K. Becker, L. N. Holzman, R. W. Lucky and E. Port, "Automatic equalization for digital communication," Proc. IEEE (Correspondence), vol. 53, January 1965, pp. 96-97.

(3) B. C. Bellows and R. S. Graham, "Experimental transversal equalizer for TD-2 radio relay system," Bell Sys. Tech. J., vol. 36, November 1957, pp. 1429-1450.

(4) Bell Telephone Laboratories, Transmission Systems for Communications, 4th ed. Winston-Salem, N.C.: Western Electric, 1970.

(5) W. R. Bennett and J. R. Davey, Data Transmission. New York: McGraw-Hill, 1965.

(6) A. D. Blumlein, H. E. Kallmann and W. S. Percival, "Electric wave filter or the like," U.S. Patent 2 263 376, April 8, 1939. Abstracted in Off. Gaz. U.S. Pat. Off., vol. 532, November 1941, pp. 706-707.

(7) W. P. Boothroyd and E. M. Creamer, Jr., "A time division multiplex system," Trans. AIEE, vol. 68, pt. 1, 1949, pp. 92-97.

(8) D. C. Coll and D. A. George, "A receiver for time-dispersed pulses," in 1965 IEEE Commun. Conv., Conf. Rec., pp. 753-757.

(9) J. Cronin-Scanlon, Advanced Calculus. Lexington, Mass.: Heath, 1969.

(10) S. De and A. C. Davies, "Convergence of adaptive equalizer for data transmission," Electronics Letters, vol. 6, December 31, 1970, pp. 858-861.

(11) M. J. DiToro, "Communication in time-frequency spread media using adaptive equalization," Proc. IEEE, vol. 56, October 1968, pp. 1653-1679.

(12) K. E. Dominiak and R. L. Pickholtz, "Finite-state equalization of a dispersive data channel in the presence of noise," in Proc. 1970 IEEE Int. Conf. Communications, paper 70-CP-318-COM.

(13) L. Fox, An Introduction to Numerical Linear Algebra. New York: Oxford U. Press, 1965.

(14) D. A. George, R. R. Bowen and J. R. Storey, "An adaptive decision feedback equalizer," IEEE Trans. Commun. Technol., vol. COM-19, June 1971, pp. 281-293.

(15) A. Gersho, "Automatic time-domain equalization with transversal filters," in Proc. 1966 Nat'l. Electron. Conf., pp. 928-932.

(16) A. Gersho, "Adaptive equalization of highly dispersive channels for data transmission," Bell Sys. Tech. J., vol. 48, January 1969, pp. 55-70.

(17) E. D. Gibson, "A highly versatile corrector of distortion and impulse noise," in Proc. 1961 Nat'l. Electron. Conf., pp. 543-556.

(18) E. Gorog, "A new approach to time-domain equalization," IBM J. Research and Develop., vol. 9, July 1965, pp. 228-232.

(19) W. J. Hemmerle, Statistical Computations on a Digital Computer. Waltham, Mass.: Blaisdell, 1967.

(20) D. Hirsch and W. J. Wolf, "A simple adaptive equalizer for efficient data transmission," IEEE Trans. Commun. Technol., vol. COM-18, February 1970, pp. 5-12.

(21) IBM Application Program H20-0205-3, System/360 Scientific Subroutine Package (360 A-CM-03X) Version III Programmer's Manual. 4th ed. (Revised February 14, 1969). White Plains, N.Y.: IBM, c1968.

(22) H. E. Kallmann, "Transversal filters," Proc. IRE, vol. 28, July 1940, pp. 302-311.

(23) V. A. Kisel, "Phase correcting circuits using delay lines," Telecommun. Radio Eng., vol. 12, pt. I, December 1965, pp. 33-40.

(24) J. J. Lang, "Delay equalizer design-numerical analysis, digital computer techniques," Doctoral dissertation, Michigan State University, East Lansing, Mich., 1961.

(25) Y. W. Lee and N. Wiener, "Electrical network system," U.S. Patent 2 128 257, July 7, 1936. Abstracted in Off. Gaz. U.S. Pat. Off., vol. 493, August 1938, pp. 1007-1008.

(26) A. Lender, "Decision-directed digital adaptive equalization technique for high-speed data transmission," IEEE Trans. Commun. Technol., vol. COM-18, October 1970, pp. 625-632.

(27) R. W. Lucky, "Automatic equalization for digital communication," Bell Sys. Tech. J., vol. 44, April 1965, pp. 547-588.

(28) R. W. Lucky, "Techniques for adaptive equalization of digital communication systems," Bell Sys. Tech. J., vol. 45, February 1966, pp. 255-286.

(29) R. W. Lucky and H. R. Rudin, Jr., "Generalized automatic equalization for communication channels," Proc. IEEE (Letters), March 1966, pp. 439-440.

(30) R. W. Lucky and H. R. Rudin, Jr., "An automatic equalizer for general-purpose communication channels," Bell Sys. Tech. J., vol. 46, November 1967, pp. 2179-2208.

(31) R. W. Lucky, J. Salz, and E. J. Weldon, Jr., Principles of Data Communication. New York: McGraw-Hill, 1968.

(32) A. H. Luyken and K. M. Miller, "A rapid method of determining the proper delay equalizers for telephone circuits," in 1965 IEEE Commun. Conv., Conf. Rec., pp. 405-408.

(33) J. Millman and H. Taub, Pulse, Digital and Switching Waveforms. New York: McGraw-Hill, 1965.

(34) C. W. Niessen, "Automatic channel equalization algorithm," Proc. IEEE (Letters), vol. 55, May 1967, p. 698.

(35) C. W. Niessen and P. R. Drouilhet, "Adaptive equalizer for pulse transmission," in 1967 IEEE Int. Commun. Conv., Conf. Rec., p. 117.

(36) C. W. Niessen and D. K. Willim, "Adaptive equalizer for pulse transmission," IEEE Trans. Commun. Technol. vol. COM-18, August 1970, pp. 377-395.

(37) M. A. Rappeport, "Automatic equalization of data transmission facility distortion using transversal equalizers," IEEE Trans. Commun. Technol., vol. COM-12, September 1964, pp. 65-73.

(38) R. B. Reynolds, "Communication Link Analyzer," Telecommunications, vol. 4, November 1970, pp. 53 and 66.

(39) H. R. Rudin, Jr., "Automatic equalization using transversal filters," IEEE Spectrum, vol. 4, January 1967, pp. 53-59.

(40) H. R. Rudin, Jr., "A continuously adaptive equalizer for general-purpose communication channels," Bell Sys. Tech. J., vol 48, July-August 1969, pp. 1865-1884.

(41) K.E. Schreiner, H. L. Funk and E. Hopner, "Automatic distortion correction for efficient pulse transmission," IBM J. Research and Develop., vol. 9, January 1965, pp. 20-30.

(42) E. D. Sunde, "Pulse transmission by AM, FM, and PM in the presence of phase distortion," Bell Sys. Tech. J., vol. 40, March 1961, pp. 353-422.

(43) P. S. Tong and B. Liu, "Automatic time-domain equalization in the presence of noise," in Proc. 1967 Nat'l. Electron. Conf., pp. 262-266.

(44) D. W. Tufts, "Nyquist's problem--The joint optimization of transmitter and receiver in pulse amplitude modulation," Proc. IEEE, vol 53, March 1965, pp. 248-259.

(45) H. A. Wheeler, "The interpretation of amplitude and phase distortion in terms of paired echoes," Proc. IRE, vol. 27, June 1939, pp. 359-385.

(46) N. Wiener and Y. W. Lee, "Electrical network system," U.S. Patent 2 124 599, July 18, 1936. Abstracted in Off. Gaz. U.S. Pat. Off., vol. 492, July 1938, p. 769.

## VII. ACKNOWLEDGEMENTS

The need for a feasibility study on this subject was indicated to the author by the late Mr. Gerald T. Bergemann of Collins Radio Company. Gratitude is expressed to the author's wife for her patience and for her typing. Computer time for this study has been provided through the Iowa State University Department of Electrical Engineering. This work has been supported in part by the Iowa State University Engineering Research Institute, project 0470.

The guidance, suggestions, and encouragement offered by the author's major professor, Dr. Robert E. Post, are especially appreciated.

## VIII. APPENDIX A: DERIVATION OF ALGORITHM

As shown in Chapter III the channel characteristic error expression is

$$E = \int_{-\infty}^{\infty} \left| \underline{X}(w) \cdot \sum_{n=-L}^{L} c_n \cdot \exp(-j\ wnT) - \underline{G}(w) \right|^2 \cdot W^2(w) \cdot dw.$$

This is expressed in terms of real and imaginary components as follows:

$$E = \int_{-\infty}^{\infty} \Big| X(w)[\cos\theta(w) - j\sin\theta(w)] \cdot \sum_n c_n[\cos(nwT) - j\sin(nwT)]$$

$$- G(w)[\cos q(w) - j\sin q(w)] \Big|^2 \cdot W^2(w) \cdot dw$$

$$= \int_{-\infty}^{\infty} \Big| [X(w) \cdot \cos\theta(w) \cdot \sum_n c_n \cos(nwT) - X(w) \cdot \sin\theta(w)$$

$$\cdot \sum_n c_n \sin(nwT) - G(w) \cdot \cos q(w)] + j[-X(w) \cdot \sin\theta(w)$$

$$\cdot \sum_n c_n \cos(nwT) - X(w) \cdot \cos\theta(w) \cdot \sum_n c_n \sin(nwT)$$

$$+ G(w)\ \sin q(w)] \Big|^2 \cdot W^2(w) \cdot dw.$$

The summation over n is implied to be from n=-L to n=+L. The magnitude squared of a complex quantity is equal to the sum of the squared real and imaginary components. In the above case the squared real component is given by

$$X^2\cos^2\theta\ (\sum_n c_n \cos nwT)^2 - X^2\cos\theta \cdot \sin\theta\ (\sum_n c_n \cos nwT)$$

$$\cdot (\sum_n c_n \sin nwT) - X \cdot G \cdot \cos\theta \cdot \cos q\ (\sum_n c_n \cos nwT) -$$

$$X^2\cos\theta\cdot\sin\theta\cdot(\sum_n c_n\cos nwT)\cdot(\sum_n c_n\sin nwT) + X^2\sin^2\theta$$
$$\cdot(\sum_n c_n\sin nwT)^2 + X\cdot G\cdot\sin\theta\cdot\cos q\ (\sum_n c_n\sin nwT) -$$
$$X\cdot G\cdot\cos\theta\cdot\cos q\ (\sum_n c_n\cos nwT) + X\cdot G\cdot\sin\theta\cdot\cos q$$
$$\cdot(\sum_n c_n\sin nwT) + G^2\cos^2 q,$$

where the notation for the w-functional dependence has been dropped for the several variables. Likewise, the squared imaginary component is given by

$$X^2\sin^2\theta\ (\sum_n c_n\cos nwT)^2 + X^2\cos\theta\cdot\sin\theta\ (\sum_n c_n\cos nwT)$$
$$\cdot(\sum_n c_n\sin nwT) - X\cdot G\cdot\sin\theta\cdot\sin q\ (\sum_n c_n\cos nwT) + X^2\cos\theta$$
$$\cdot\sin\theta\ (\sum_n c_n\cos nwT)\cdot(\sum_n c_n\sin nwT) + X^2\cos^2\theta\ (\sum_n c_n\sin nwT)^2$$
$$- X\cdot G\cdot\cos\theta\cdot\sin q\ (\sum_n c_n\sin nwT) - X\cdot G\cdot\sin\theta\cdot\sin q$$
$$\cdot(\sum_n c_n\cos nwT) - X\cdot G\cdot\cos\theta\ \sin q\ (\sum_n c_n\sin nwT) + G^2\sin^2 q.$$

When these expressions are summed together certain terms cancel immediately and the resultant is

$$X^2(\cos^2\theta + \sin^2\theta)[(\sum_n c_n\cos nwT)^2 + (\sum_n c_n\sin nwT)^2] +$$
$$G^2(\cos^2 q + \sin^2 q) - 2\cdot X\cdot G\cdot\cos q\ [\cos\theta\ (\sum_n c_n\cos nwT) -$$
$$\sin\theta\ (\sum_n c_n\sin nwT)] - 2\cdot X\cdot G\cdot\sin q\ [\cos\theta\ (\sum_n c_n\sin nwT)$$
$$+ \sin\theta\ (\sum_n c_n\cos nwT)].$$

After utilizing the trigonometric identity, $\cos^2 u + \sin^2 u = 1$, and arranging terms, the above is equal to

$$X^2[(\sum_n c_n \cos nwT)^2 + (\sum_n c_n \sin nwT)^2] + G^2 - 2 \cdot X \cdot G \cdot \cos q$$

$$\cdot \left|\sum_n c_n[(\cos nwT) \cos \theta - (\sin nwT) \sin \theta]\right| - 2 \cdot X \cdot G \cdot \sin q$$

$$\cdot \left|\sum_n c_n[(\sin nwT) \cos \theta + (\cos nwT) \sin \theta]\right|.$$

By using the trigonometric identities, $\cos u \cdot \cos v \mp \sin u \cdot \sin v = \cos(u \pm v)$ and $\sin u \cdot \cos v + \cos u \cdot \sin v = \sin(u+v)$, the following equivalent expressions for the above are obtained:

$$X^2[(\sum_n c_n \cos nwT)^2 + (\sum_n c_n \sin nwT)^2] + G^2 - 2 \cdot X \cdot G \cdot \cos q$$

$$\cdot \sum_n c_n \cos(nwT+\theta) - 2 \cdot X \cdot G \cdot \sin q \cdot \sum_n c_n \sin(nwT+\theta)$$

$$= X^2[(\sum_n c_n \cos nwT)^2 + (\sum_n c_n \sin nwT)^2] + G^2 - 2 \cdot X \cdot G$$

$$\cdot \sum_n c_n[\cos q \cdot \cos(nwT+\theta) + \sin q \cdot \sin(nwT+\theta)]$$

$$= X^2[(\sum_n c_n \cos nwT)^2 + (\sum_n c_n \sin nwT)^2] + G^2 - 2 \cdot X \cdot G$$

$$\cdot \sum_n c_n \cos(q-nwT-\theta).$$

Therefore, the channel characteristic error is

$$E = \int_{-\infty}^{\infty} \left| X^2[(\sum_n c_n \cos nwT)^2 + (\sum_n c_n \sin nwT)^2] + G^2 - 2 \cdot X \cdot G \cdot \sum_n c_n \cos(q-nwT-\theta)\right| W^2 \cdot dw.$$

If a minimum error with respect to each tap gain, $c_k$, exists it must occur when the corresponding partial derivative is zero, that is when

$$\frac{\partial E}{\partial c_k} = \int_{-\infty}^{\infty}\Big( X^2[2(\cos kwT)\cdot\sum_n c_n \cos nwT + 2(\sin kwT)\cdot\sum_n c_n \cdot\sin nwT] + 0 - 2\cdot X\cdot G\cdot\cos(q-kwT-\theta)\Big)W^2\cdot dw = 0,$$

for $k = -L,-L+1,\ldots,L-1,L$.

The differentiation inside the integrand is permitted by Leibniz' rule [9] since all of the functions and their derivatives are continuous in $c_k$. The terms within the inner brackets can be expressed as

$$(\cos kwT)\cdot\sum_n c_n \cos nwT + (\sin kwT)\cdot\sum_n c_n \sin nwT$$

$$= \tfrac{1}{2}\cdot\sum_n c_n[\cos(nwT+kwT) + \cos(nwT-kwT)] + \tfrac{1}{2}\cdot\sum_n c_n[\cos(nwT-kwT) - \cos(nwT+kwT)]$$

$$= \sum_n c_n \cos(nwT-kwT) = \sum_n c_n[(n-k)wT].$$

Utilizing this result in the expression for $\frac{\partial E}{\partial c_k}$ above yields, after division through by 2,

$$0 = \int_{-\infty}^{\infty}\Big( X^2\cdot\sum_n c_n \cos[(n-k)wT] - X\cdot G\cdot\cos(q-kwT-\theta)\Big)W^2\cdot dw.$$

Since k is successively assigned each integer value between -L and +L there are 2L+1 simultaneous equations. The functions in the integrand all possess even symmetry about zero frequency so that the lower limit can be replaced by zero.

In the equalization problem under consideration the value of $X(w)$ and $\theta(w)$ is assumed to be specified at M equally spaced discrete frequencies only. Thus each integral equation is replaced by a summation equation having the following form:

$$0 = \sum_{i=1}^{M} \Big( X^2(w_i)\cdot\sum_n c_n \cos[(n-k)w_iT] - X(w_i)\cdot G(w_i)$$
$$\cdot\cos[q(w_i) - \theta(w_i) - kw_iT]\Big)\cdot W^2(w_i)\cdot \Delta w,$$

where $\Delta w$ = constant.

After division through by $\Delta w$ the k-th equation can be rewritten as

$$\sum_i X^2(w_i)\cdot W^2(w_i)\cdot\sum_n c_n \cos[(n-k)w_iT]$$
$$= \sum_i X(w_i)\cdot G(w_i)\cdot W^2(w_i)\cdot\cos[q(w_i)-\theta(w_i)-kw_iT] \equiv b_k.$$

The right-hand side defines the k-th element of the constant vector. The left-hand side summations over n and i can be written out explicitly and then terms regrouped to yield the k-th equation:

$$c_{-L}\cdot\sum_i X^2(w_i)\cdot W^2(w_i)\cdot\cos[(-L-k)w_iT] + \dots$$
$$+ c_0\cdot\sum_i X^2(w_i)\cdot W^2(w_i)\cdot\cos[(0-k)w_iT] + \dots$$
$$+ c_L\cdot\sum_i X^2(w_i)\cdot W^2(w_i)\cdot\cos[(L-k)w_iT] = b_k.$$

The system of equations is put into matrix form by first defining

$$a_{k,n} \equiv \sum_i X^2(w_i)\cdot W^2(w_i)\cdot \cos[(n-k)w_i T],$$

the coefficient of $c_n$ in the k-th equation. Then the k-th equation becomes

$$a_{k,-L}\cdot c_{-L} + \ .\ .\ . + a_{k,o}\cdot c_o + \ .\ .\ . + a_{k,L}\cdot c_L = b_k.$$

Hence the N simultaneous equations in the N unknowns (the tap gains, $c_n$) are summarized in the matrix equation,

$$A\cdot\overline{c} = \overline{b}\ .$$

The dependence of $a_{k,n}$ upon k and n is only with respect to the difference, (n-k). The matrix is symmetric since the cosine is an even function. Hence it suffices to compute only the N distinct coefficients and then "fill-in" the matrix using these values. The form of the matrix thus becomes

$$A = \begin{bmatrix} a_o & a_1 & a_2 & \cdots & a_{N-2} & a_{N-1} \\ a_1 & a_o & a_1 & \cdots & a_{N-3} & a_{N-2} \\ . & . & . & . & . & . \\ a_{N-1} & a_{N-2} & a_{N-3} & \cdots & a_1 & a_o \end{bmatrix}$$

where the coefficient subscript identifies the quantity $|n-k|$. This property is utilized in the computer program algorithm.

# IX. APPENDIX B: EQUALIZED CHANNEL ERROR EXPRESSION

To obtain the total unnormalized error, E, the incremental error to be evaluated at each of the 2M frequencies is

$$e = \left|\underline{H}(w) - \underline{G}(w)\right|^{2} \cdot W^{2}(w) \cdot dw.$$

By the definitions of Chapter III the equalized channel characteristic is

$$\begin{aligned}\underline{H}(w) &= H(w) \cdot \exp(-j\ (\theta(w)+\phi(w))\\ &= H(w)\left[\cos(-\theta(w)-\phi(w)) + j\ \sin(-\theta(w)-\phi(w))\right].\end{aligned}$$

For this study the ideal channel characteristic is

$$\underline{G}(w) = 1.0 \cdot \exp(-j\ p \cdot w) = \cos(-p \cdot w) + j\ \sin(-p \cdot w).$$

The magnitude squared of a complex quantity is the sum of the real and imaginary component squares; therefore,

$$e = \left( \left[H \cdot \cos(\theta+\phi) - \cos(p \cdot w)\right]^2 + \left[-H \cdot \sin(\theta+\phi) + \sin(p \cdot w)\right]^2 \right) \cdot W^2 \cdot dw,$$

dropping the w-dependence notation and utilizing standard trigonometric identities.

The error reference value chosen for this study is determined by assuming that the reference channel has a perfect amplitude response, $H(w)=1$, and that a unity error weighting for all frequencies is employed, requiring $W^2(w)=1$. The incremental reference error is given by

$$e_{ref} = \left([1\cdot\cos(\theta+\phi) - \cos(p\cdot w)]^2 + [-1\cdot\sin(\theta+\phi) + \sin(p\cdot w)]^2\right)$$

$$\cdot dw.$$

Using additional identities this becomes

$$e_{ref} = [\cos^2(\theta+\phi) - 2\cdot\cos(\theta+\phi)\cdot\cos(p\cdot w) + \cos^2(p\cdot w) +$$

$$\sin^2(\theta+\phi) - 2\cdot\sin(\theta+\phi)\cdot\sin(p\cdot w) + \sin^2(p\cdot w)]\cdot dw$$

$$= 2\left(1 - [\cos(\theta+\phi)\cdot\cos(p\cdot w) + \sin(\theta+\phi)\cdot\sin(p\cdot w)]\right)\cdot dw,$$

or

$$e_{ref}(z) = 2(1 - \cos z)\cdot dw,$$

defining the phase error as $z(w) = [\theta(w)+\phi(w)] - p\cdot w$.

Assume that over the bandwidth of interest the probability of z is uniform between 0 and $2\pi$ radians, requiring a probability density of $1/2\pi$. Then the expected value of the incremental reference error is

$$\overline{e_{ref}} = \int_0^{2\pi} e_{ref}(z)\cdot(\text{probability of } z)\cdot dz$$

$$= [\int_0^{2\pi} 2(1-\cos z)\cdot(1/2\pi)\cdot dz]\cdot dw = 2\cdot dw.$$

The total reference error over the bandwidth, BW Hz, considering both positive and negative frequencies is given by

$$E_{REF} = 2\cdot\frac{2\pi\cdot BW}{dw}\cdot\overline{e_{ref}} = 8\pi\cdot BW.$$

The weighting-area correction factor is to be the weighting function average over the bandwidth. It is computed by

$$A_{eq} = \frac{1}{2M} \cdot \sum_{i=1}^{2M} W_i^2 .$$

The expression for the (normalized) equalized channel mean-square error is therefore

$$\frac{E}{E_{REF}} \cdot \frac{1}{A_{eq}} = \frac{E}{8\pi \cdot BW \cdot A_{eq}} .$$

Thus the equalized channel root-mean-square error expression in terms of E is $[E/(8\pi \cdot BW \cdot A_{eq})]^{\frac{1}{2}}$. This is converted to percent (of the reference) by multiplication by 100.

# X. APPENDIX C: PROGRAM FLOW DIAGRAM AND LISTING

Figure 18 is the simplified flow diagram for program EQLB, the simulation model. It is followed by the program listing.

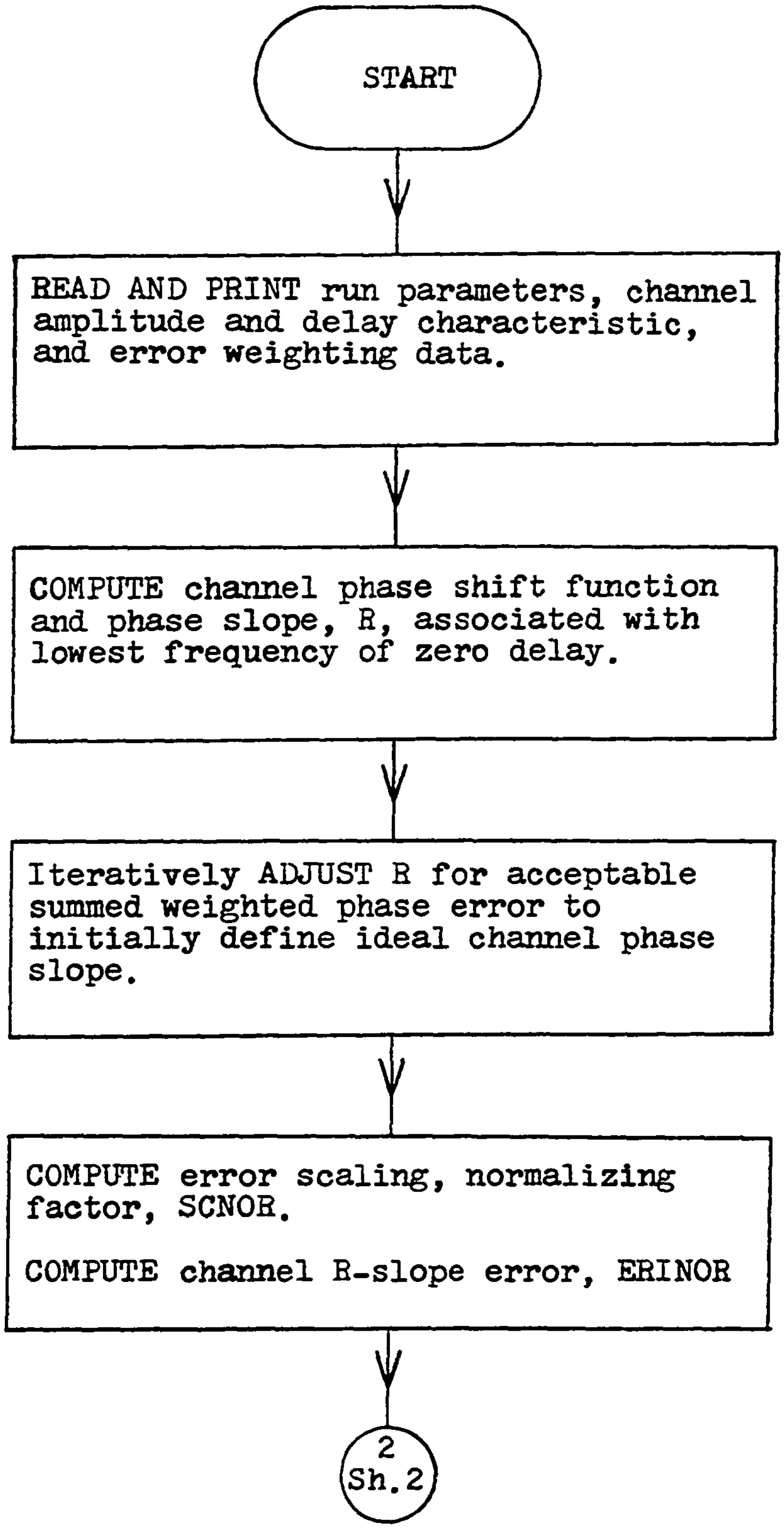


Figure 18. Program EQLR flow diagram Sh. 1 of 9

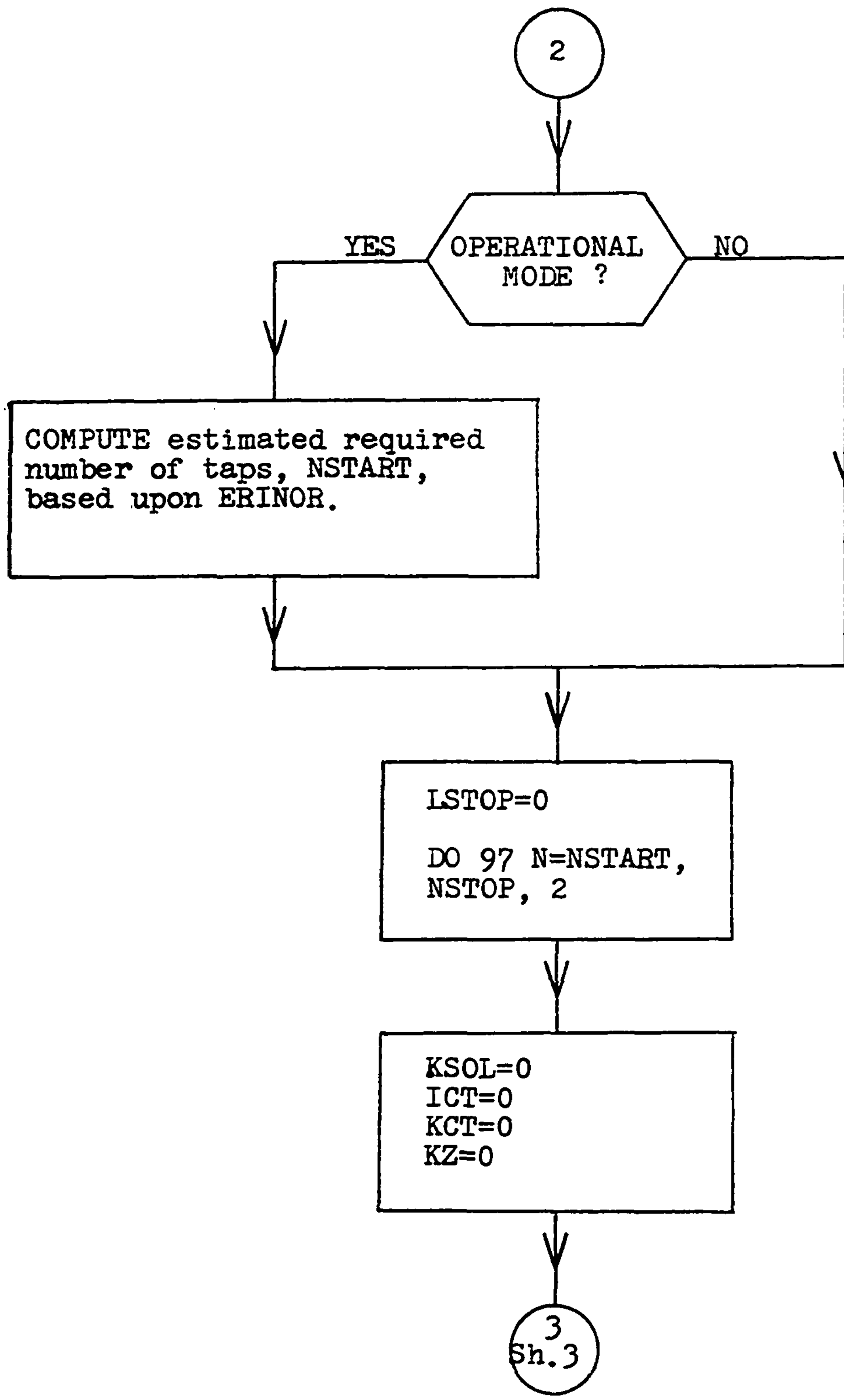


Figure 18 continued Sh. 2 of 9

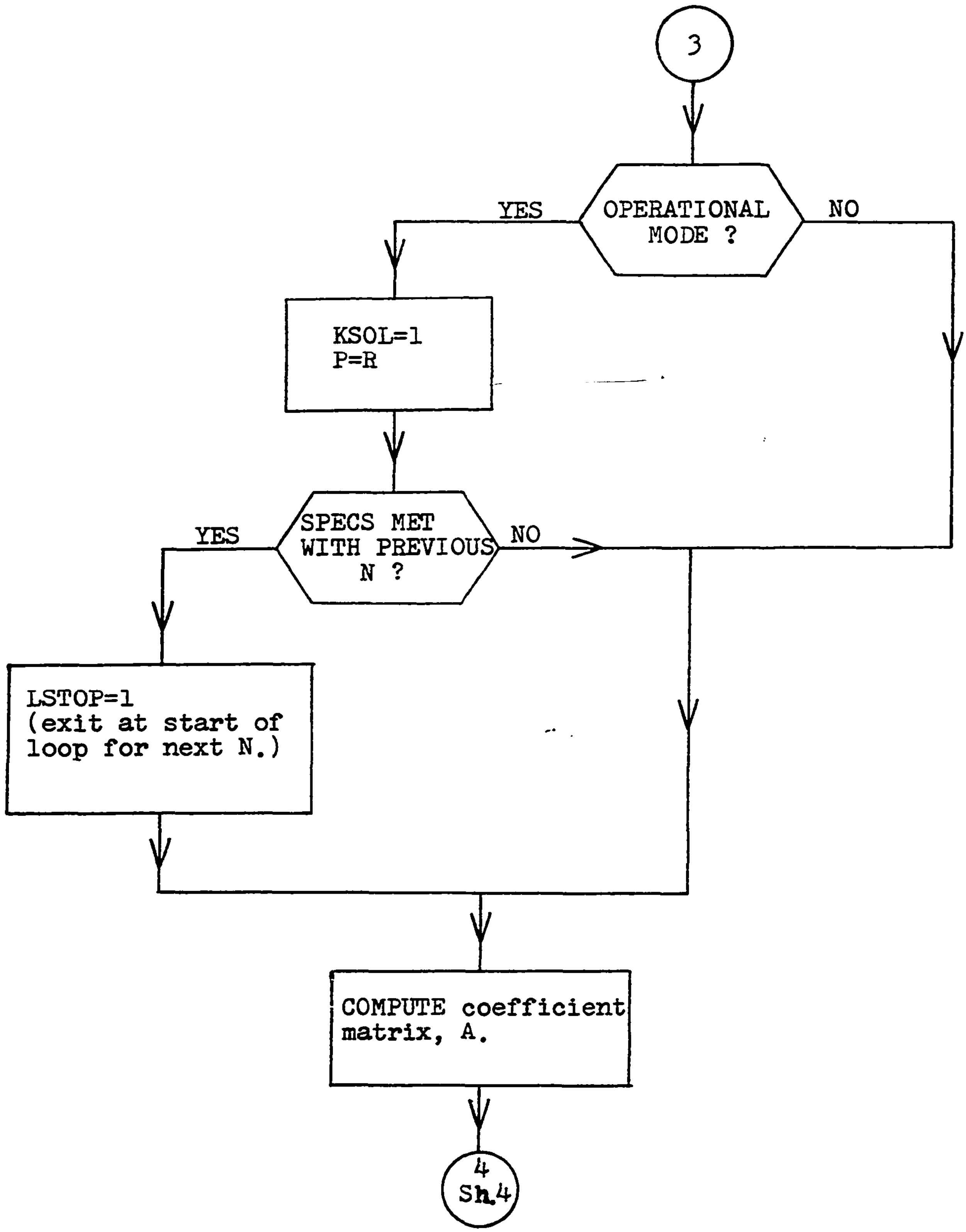


Figure 18 continued Sh. 3 of 9

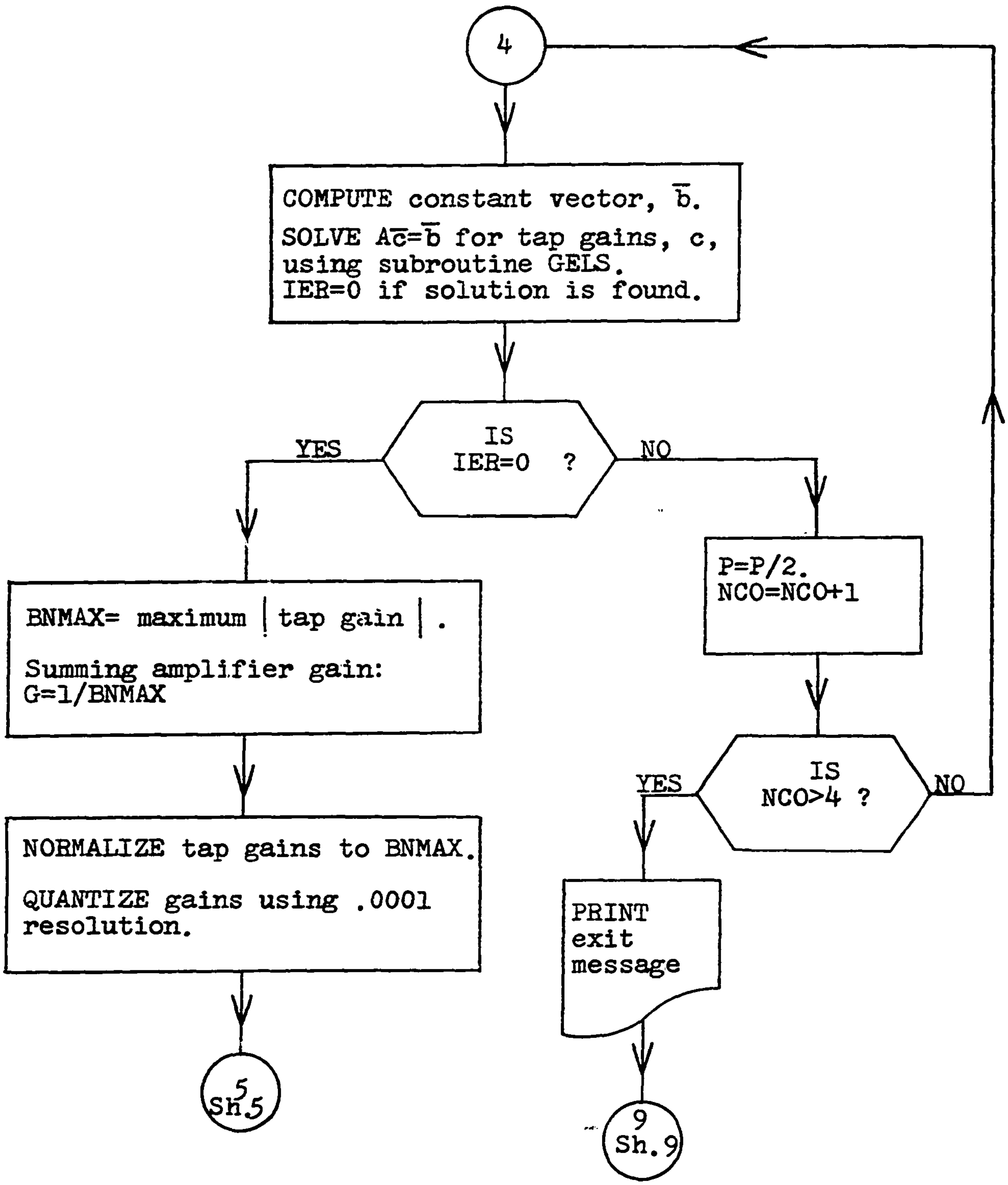


Figure 18 continued Sh. 4 of 9

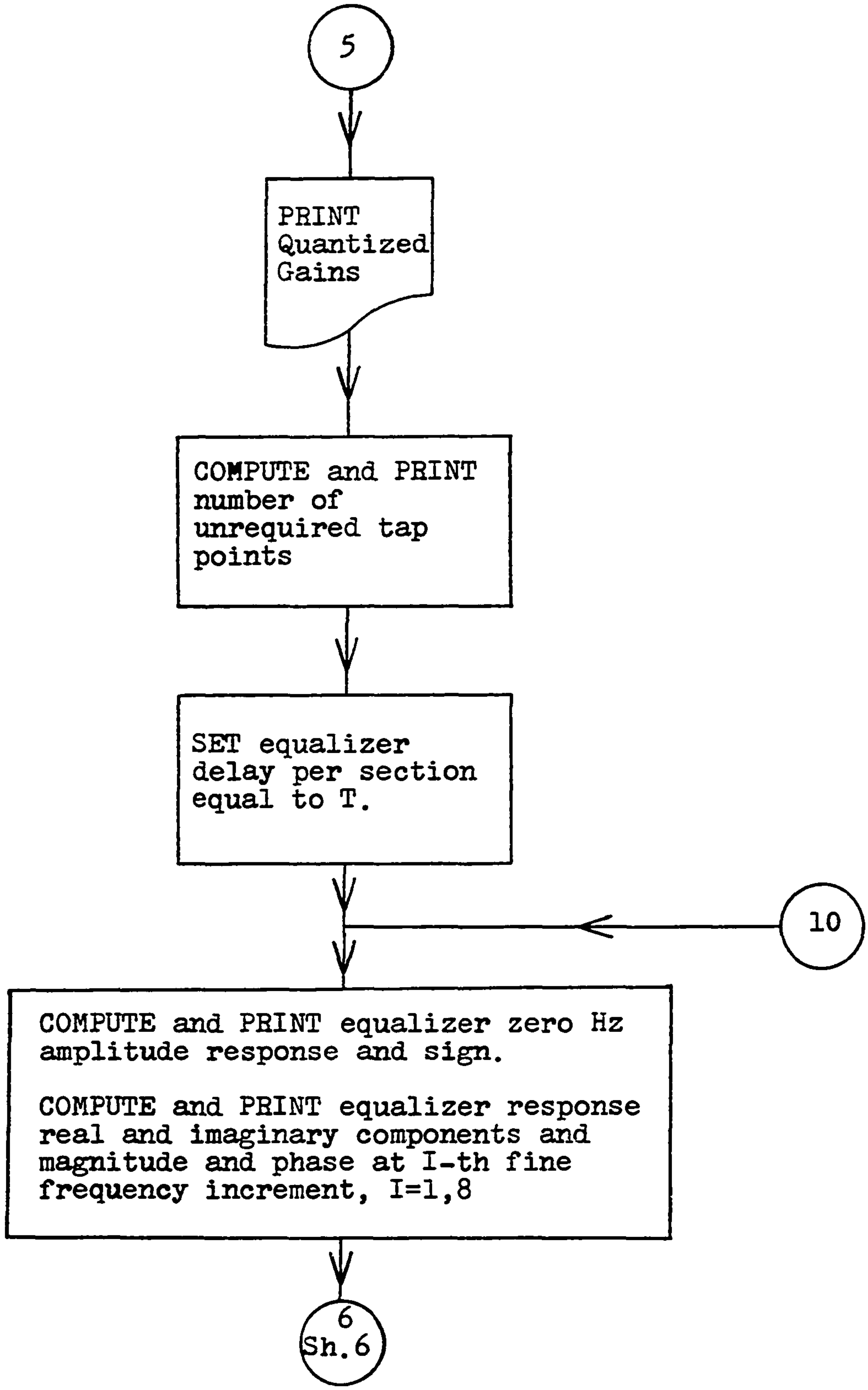


Figure 18 continued Sh. 5 of 9

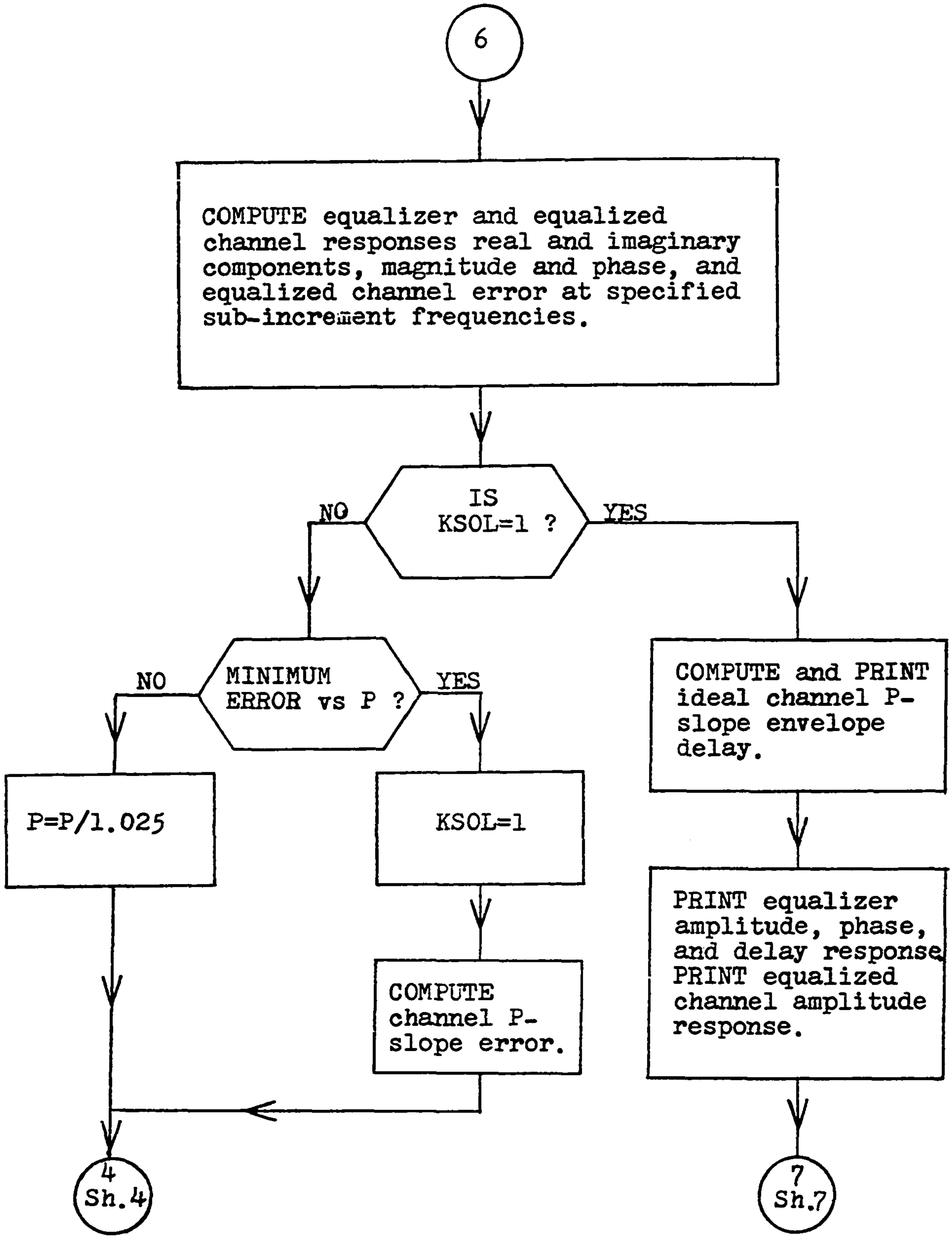


Figure 18 continued Sh. 6 of 9

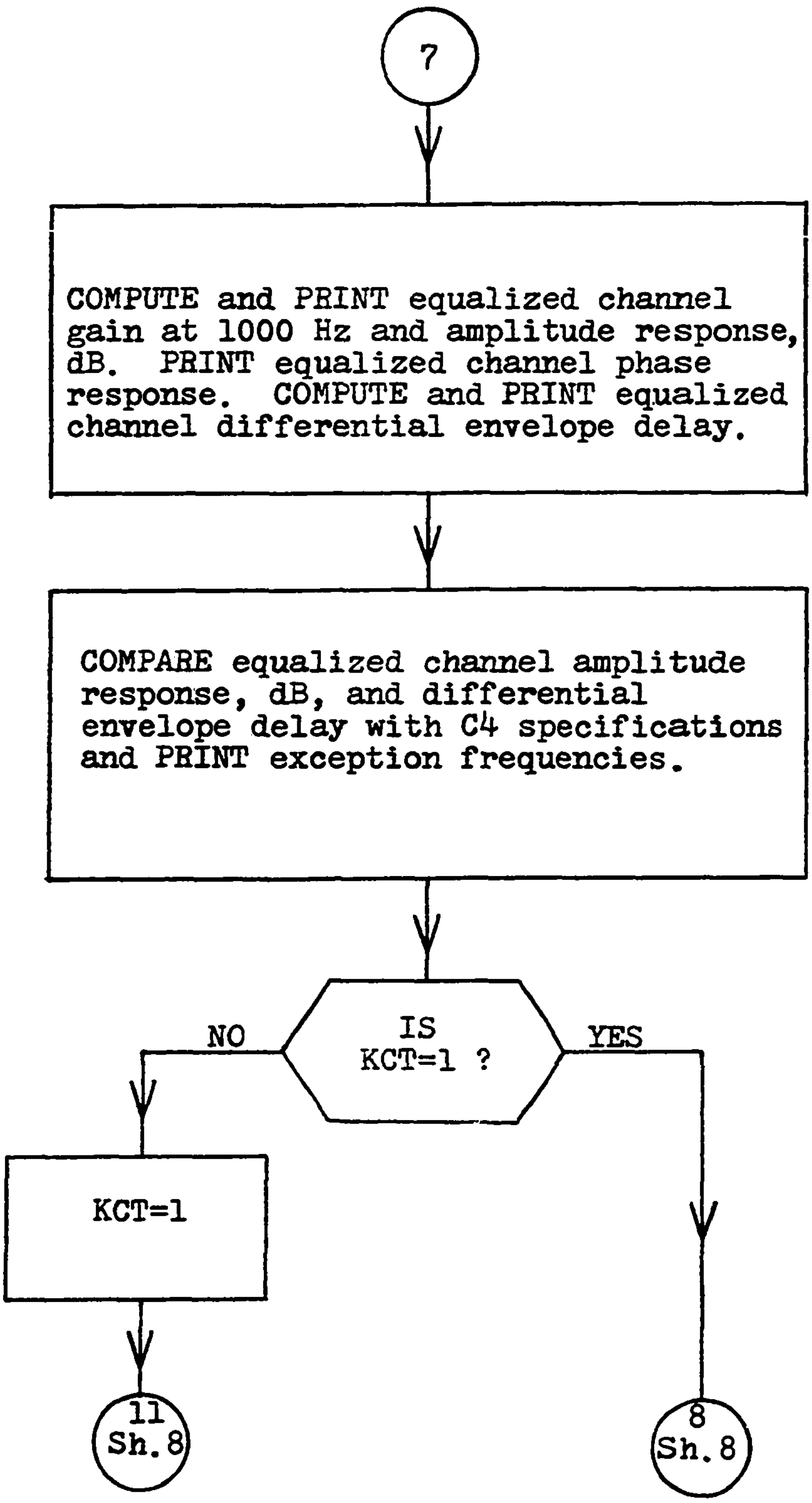


Figure 18 continued Sh. 7 of 9

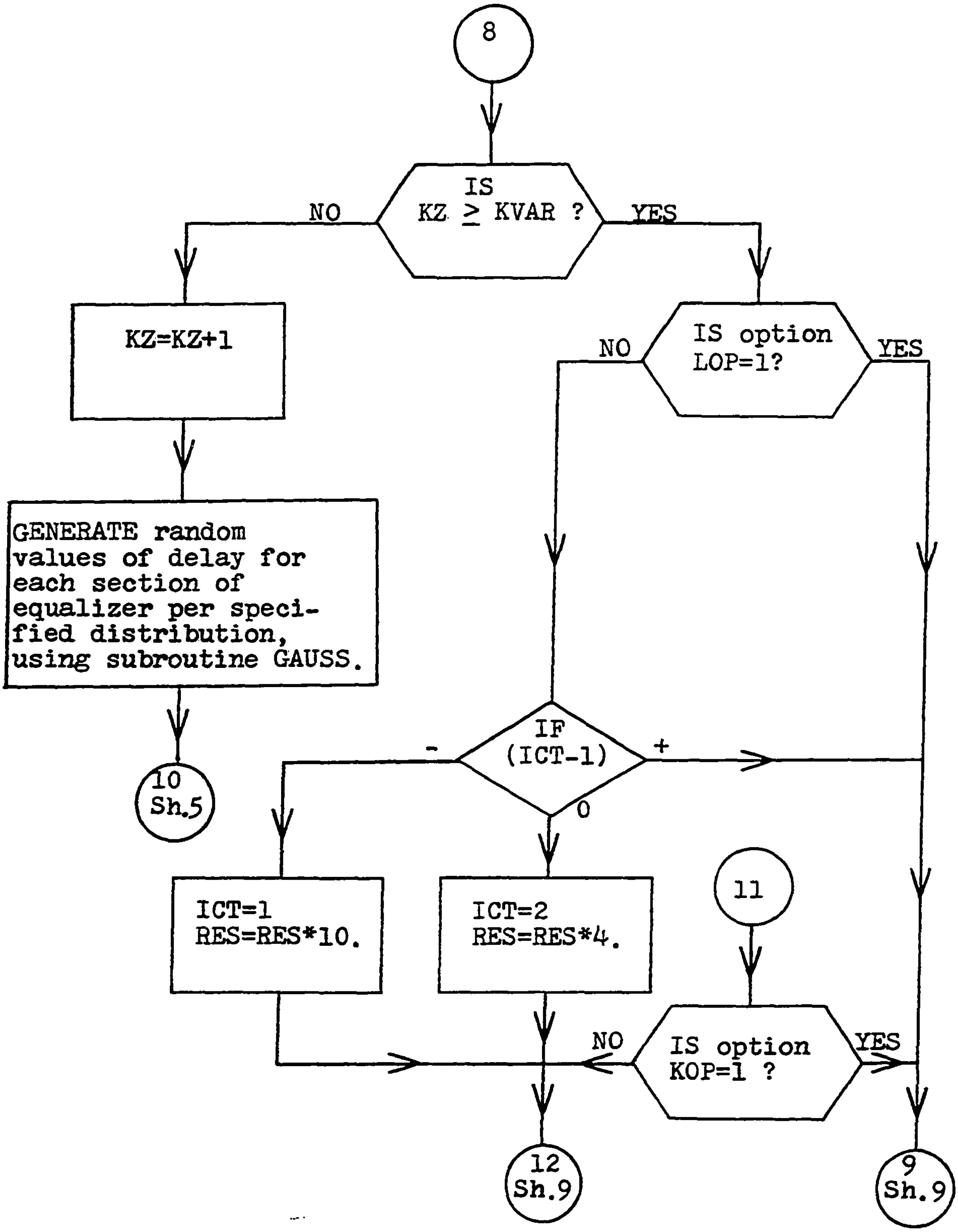


Figure 18 continued Sh. 8 of 9

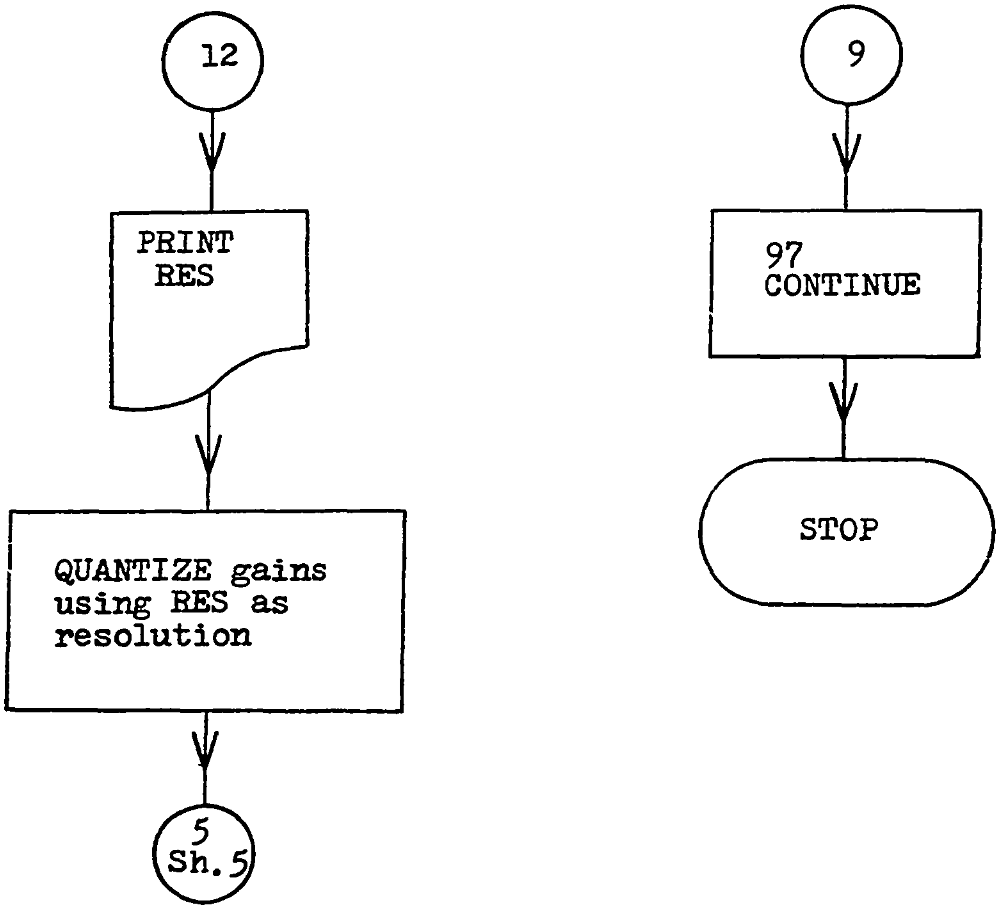


Figure 18 continued Sh. 9 of 9

```
C         TAPPED DELAY LINE EQUALIZER SIMULATION PROGRAM, EQLR, 5-13-71.
C         WRITTEN BY DAVID H. BLISS
C         AUTOMATIC AMPLITUDE AND DELAY EQUALIZATION DERIVED FROM
C         FREQUENCY-DOMAIN DATA.
      DIMENSION X(71),D(70),TH(70),W(70),CR(140),CI(140),CM(70),CA(70),
     1HM(70),HA(70),A(630),B(35),AA(35),Y(35),AUX(34),E(70)
C         NRUN=NO. OF RUNS IN THIS JOB
C         IXE=STARTING INTEGER FOR RANDOM NUMBER GENERATOR
      READ(1,60) NRUN,IXE
   60 FORMAT(I2,1X,I9)
      DO 100 MRUN=1,NRUN
C         M=NO. OF FREQUENCY LOCATIONS WHERE CHANNEL DATA GIVEN (35 MAX)
C         NSTART=LOWEST NUMBER OF TAP POINTS CONSIDERED (3 MINIMUM)
C         NSTOP=HIGHEST NUMBER OF TAP POINTS CONSIDERED (35 MAXIMUM)
C         NOTE* THESE VALUES ARE REVISED INTERNALLY IN OPERATIONAL MODE
C
C         KVAR=NO. OF MONTE-CARLO EQUALIZER-DELAY-VARIATION SUB-RUNS
C         F=CHANNEL DATA FREQUENCY INCREMENT, HZ
C         NOTE* CHANNEL DATA MUST BE GIVEN AT EACH (I*F) HZ LOCATION,
C               WHERE I=1,2,...,M
C         T=EQUALIZER DELAY PER SECTION, SECONDS
C         RES=TAP GAINS AND SUMMING AMPLIFIER GAIN RESOLUTION
C         AM=MEAN OF EQUALIZER DELAY DISTRIBUTION (1.0 IS NOMINAL)
C         SD=STANDARD DEVIATION OF EQUALIZER DELAY DISTRIBUTION
C         REJ=LARGEST USABLE EQUALIZER DELAY VARIATION FROM THE MEAN
C         KOP=0, MAKE SUB-RUN USING RES FOR RESOLUTION. NOP=1 SETS KOP=0
C            =1, OMIT ABOVE SUB-RUN, DELAY VARIATION AND ADDITIONAL RES
C         LOP=0, MAKE SUB-RUNS USING RES*10 AND RES*40 FOR RESOLUTION
C            =1, OMIT THESE SUB-RUNS. NOP=1 SETS LOP=1
C         NOP=0, INVESTIGATION MODE
C            =1, OPERATIONAL MODE
      READ(1,61) M,NSTART,NSTOP,KVAR,F,T,RES,AM,SD,REJ,KOP,LOP,NOP
   61 FORMAT(4(I2,1X),F5.1,1X,F7.6,1X,F5.4,3(1X,F4.2),3I2)
      M2=2*M
      M4=4*M
      RESOR=RES
      DW=3.141593*F/4.0
```

```
      DS=6.283185*F*T/8.C
      A1=1.570796
      A2=3.141593
      WRITE(3,2) MRUN,NSTART,NSTOP,KVAR,M,F,T,RES,AM,SD,REJ,IXE,KOP,LOP
    2 FORMAT('1',' RUN NUMBER',I2,5X,'NSTART=',I2,5X,'NSTOP=',I2,5X,'KVA
     1R=',I2,/,'0','NO. OF FREQ. LOCATIONS=',I2,5X,'FREQ. INCREMENT=',F6
     2.1,' HZ',/,'0',' DELAY PER SECTION=',F8.6,' SECONDS',5X,' GAIN RES
     3OLUTION=',F7.5,/,'0',' TAP DISTRIBUTION MEAN=',F5.2,5X,'STD.DEV.='
     4,F5.2,5X,'ACCEPTANCE TOL.=',F5.2,5X,'IXE=',I9,5X,'KOP=',I2,5X,'LOP
     5=',I2)
      IF(NOP.EQ.1) GO TO 1000
      WRITE(3,1001)
 1001 FORMAT('0',//,'0',' INVESTIGATION MODE (NOP=0)',//,'0  ')
      GO TO 1002
 1000 WRITE(3,1003)
 1003 FORMAT('0',//,'0',' OPERATIONAL MODE (NOP=1)',//,'0  ')
 1002 CONTINUE
C          IF F.LT.200 HZ, EACH FREQ. INCREMENT IS INTERNALLY DIVIDED
C          INTO 4 SUB-INCREMENTS.  OTHERWISE, 8 SUB-INCREMENTS ARE USED.
C          IN EITHER CASE THE ERROR WEIGHTING, W( ), MUST BE GIVEN AT F/2
C          HZ INCREMENTS, AND THE CHANNEL AMPLITUDE RESPONSE, X( ), AND
C          DELAY, D( ), MUST BE GIVEN AT F HZ INCREMENTS. X IS DIMENSION-
C          LESS VOLTAGE RATIO AND D IS IN SECONDS, ALL NON-NEGATIVE.
C          PHASE SHIFT, TH( ) RADIANS, IS COMPUTED BY INTEGRATING DELAY.
C          NOTE* IF THE NUMBER OF W VALUES OR X VALUES IS AN INTEGER (4
C                OR LESS) MULTIPLE OF 16, THE RESPECTIVE DATA CARD SET
C                MUST BE FOLLOWED BY A BLANK CARD. IF THE NUMBER OF D
C                VALUES IS AN INTEGER (3 OR LESS) MULTIPLE OF 11, THE
C                DELAY DATA CARD SET MUST BE FOLLOWED BY A BLANK CARD.
C
C
C          THE FOLLOWING ASSUMPTIONS ARE MADE FOR THIS PROGRAM.
C
C          IDEAL CHANNEL AMPLITUDE RESPONSE=1.0 FOR ALL FREQUENCIES.
C          IDEAL CHANNEL DIFFERENTIAL DELAY=CONSTANT FOR ALL FREQUENCIES.
C
C
```

```
      IF(F.GE.200.) GO TO 999
      READ(1,62) (W(IW),IW=1,M2)
   62 FORMAT(16F5.2,/,16F5.2,/,16F5.2,/,16F5.2,/,16F5.2)
      READ(1,62) (X(IX),IX=2,M2,2)
      READ(1,63) (D(ID),ID=2,M2,2)
   63 FORMAT(11F7.5,/,11F7.5,/,11F7.5,/,11F7.5)
      X(1)=X(2)/2.
      X(M2+1)=X(M2)/2.0
      TH(1)=D(2)*3.141593*F
      TH(2)=D(2)*6.283185*F
      DO 3 I=4,M2,2
    3 TH(I)=TH(I-2)+3.141593*F*(D(I-2)+D(I))
      M2M=M2-1
      DO 4 I=3,M2M,2
      TH(I)=(TH(I-1)+TH(I+1))/2.
    4 X(I)=(X(I-1)+X(I+1))/2.
      LAM=1
      LIM=2*M
      LUM=LIM-1
      MSF=2
      Q=0.25
      GO TO 29
  999 READ(1,62) (W(IW),IW=2,M4,2)
      READ(1,62) (X(IX),IX=4,M4,4)
      READ(1,63) (D(ID),ID=4,M4,4)
      X(2)=X(4)/2.
      X(M4+2)=X(M4)/2.
      TH(1)=D(4)*1.570796*F
      TH(2)=D(4)*3.141593*F
      TH(3)=D(4)*4.712389*F
      TH(4)=D(4)*6.283185*F
      DO 903 IQ=8,M4,4
  903 TH(IQ)=TH(IQ-4)+3.141593*F*(D(IQ-4)+D(IQ))
      M4M=M4-2
      DO 904 IP=6,M4M,4
      TH(IP)=(TH(IP-2)+TH(IP+2))/2.
  904 X(IP)=(X(IP-2)+X(IP+2))/2.
```

```
      M4S=M4-1
      DO 906 IS=5,M4S,2
  906 TH(IS)=(TH(IS-1)+TH(IS+1))/2.
      LAM=2
      LIM=4*M
      LUM=LIM-1
      MSF=4
      Q=0.5
C           THE LOWEST FREQUENCY OF ZERO DELAY IS IDENTIFIED AS NSL.
   29 DO 30 JK=MSF,LIM,MSF
      IF(D(JK)) 30,31,30
   31 NSL=JK
      GO TO 32
   30 CONTINUE
   32 R=TH(NSL)/NSL
C           R IS PROPORTIONAL TO THE ZERO INTERCEPT PHASE SLOPE ASSOCIATED
C           WITH TH(NSL).
      WRITE(3,33) NSL,TH(NSL)
   33 FORMAT('0',' ZERO DELAY FREQ. COUNT=',I3,'  PHASE HERE=',F14.6,' R
     8ADIANS')
C           FREQUENCY INTEGERS FOR 1000 HZ AND 2600 HZ ARE IDENTIFIED.
      DO 400 NT=1,LIM
      IF(NT*F/MSF.LT.999.) GO TO 400
      NL=NT
      GO TO 401
  400 CONTINUE
      NL=8
  401 DO 402 NZ=1,LIM
      IF(NZ*F/MSF.LE.2601.) GO TO 402
      NH=NZ-1
      GO TO 403
  402 CONTINUE
      NH=LIM-1
  403 CONTINUE
C           EPS DEFINES THE MINIMUM MODULUS OF A PIVOT ELEMENT (DIVISOR)
C           IN THE SIMULTANEOUS EQUATION SOLUTION SUBROUTINE, GELS.
      EPS=.000001
```

```
      NCOL=1
      WRITE(3,64)
   64 FORMAT('0',' R,SLOPE   PH. PT.   SUMPHE')
C          R IS ITERATIVELY ADJUSTED UNTIL THE WEIGHTED PHASE DIFFERENCE
C          (BETWEEN THE CHANNEL PHASE AND THE R-SLOPE PHASE) SUMMATION,
C          SUMPHE, IS WITHIN THE INTERVAL, ZERO +/- TOLE.
C          THIS INITIALLY DEFINES THE IDEAL CHANNEL DELAY.
      TOLE=10.
      NNEG=0
      NPOS=0
      ISC=1
      IR=0
   51 SUMPHE=0.0
      IR=IR+1
      DO 50 K=LAM,LIM,LAM
   50 SUMPHE=SUMPHE+(TH(K)-R*K)*W(K)
      PHPT=R*NSL
      WRITE(3,65) R,PHPT,SUMPHE
   65 FORMAT(' ',F7.4,2X,F8.4,2X,F9.3)
      IF(ABS(SUMPHE).LE.TOLE) GO TO 56
      IF(R.EQ.0.0) GO TO 56
      IF(IR.GT.50) GO TO 56
      IF(SUMPHE) 52,56,53
   52 IF(NPOS.LT.1) GO TO 54
      IF(NNEG.LT.1) GO TO 54
      ISC=ISC+1
   54 R=(1.-.15/ISC)*R
      NNEG=1
      GO TO 51
   53 IF(NNEG.LT.1) GO TO 55
      IF(NPOS.LT.1) GO TO 55
      ISC=ISC+1
   55 R=(1.+.175/ISC)*R
      NPOS=1
      GO TO 51
   56 CONTINUE
      WRITE(3,67)
```

```
   67 FORMAT('0',' CHANNEL AMPLITUDE CHARACTERISTIC')
      WRITE(3,68) (X(LX),LX=LAM,LIM,LAM)
   68 FORMAT(' ',5(/,' ',14F9.3))
      WRITE(3,69)
   69 FORMAT('0',' CHANNEL DELAY CHARACTERISTIC')
      WRITE(3,70) (D(LD),LD=MSF,LIM,MSF)
   70 FORMAT(' ',4(/,' ',10F12.6))
      WRITE(3,71)
   71 FORMAT('0',' CHANNEL PHASE CHARACTERISTIC (DERIVED)')
      WRITE(3,68) (TH(LTH),LTH=1,LIM)
      WRITE(3,72)
   72 FORMAT('0',' ERROR WEIGHTING CHARACTERISTIC')
      WRITE(3,68) (W(LW),LW=LAM,LIM,LAM)
C          THE CHANNEL ERROR ASSOCIATED WITH R IS COMPUTED.
      ERINIT=0.0
      DO 35 K=LAM,LIM,LAM
      ERINIT=ERINIT+((X(K)*COS(TH(K))-COS(R*K))**2+(X(K)*SIN(TH(K))-SIN(
     1R*K))**2)*W(K)*6.283185*F
   35 CONTINUE
      IF(F.GE.200.) GO TO 997
      S=3.141593*F*T
      GO TO 998
  997 S=1.570796*F*T
  998 FF=S*MSF*M/(6.283185*T)
      WRITE(3,75) FF
   75 FORMAT('0',' HIGHEST FREQUENCY=',F10.3,' HZ')
C          THE ERROR SCALING, NORMALIZING FACTOR, SCNOR, IS COMPUTED.
      WTSUM=0.0
      DO 1004 I=LAM,LIM,LAM
 1004 WTSUM=WTSUM+W(I)
      AEQ=WTSUM/M2
      SCNOR=.2/(SQRT(M*F*AEQ))
      WRITE(3,1005) AEQ,SCNOR
 1005 FORMAT('0',' EQUIVALENT AREA WEIGHT=',E11.5,'   ERROR SCALING, NOR
     1MALIZING FACTOR=',E11.5)
      ERINOR=SCNOR*SQRT(ERINIT)
      ERINPC=100.*ERINOR
```

```
      WRITE(3,1006) ERINIT,ERINOR,ERINPC
 1006 FORMAT('0',' CHANNEL R-SLOPE RAW SQUARED ERROR=',E11.5,'   SCALED,
     1 NORMALIZED ERROR=',E11.5,'   IN PERCENT=',F7.2)
      IF(NOP.NE.1) GO TO 1101
C          THE REQUIRED NUMBER OF TAP POINTS IS ESTIMATED BASED UPON THE
C          CHANNEL R-SLOPE ERROR. (*OPERATIONAL MODE ONLY*)
      KOP=0
      LOP=1
      NSTOP=35
      NIT=ERINOR*5.01
      NSTART=NIT*2.01+1.
      IF(NSTART.GE.3) GO TO 1100
      IF(ERINOR.GE.0.07) GO TO 1099
      WRITE(3,1098)
 1098 FORMAT('0',' NO EQUALIZER REQUIRED')
      GO TO 100
 1099 NSTART=3
 1100 IF(NSTART.LE.35) GO TO 1096
      NSTART=35
 1096 WRITE(3,1102) NSTART
 1102 FORMAT('0',' OPERATIONAL MODE.  ESTIMATED REQUIRED NUMBER OF TAP P
     1OINTS=',I3)
 1101 CONTINUE
      JSTOP=0
      KSTOP=0
      LSTOP=0
C          MAIN EQUALIZER SOLUTION AND PERFORMANCE EVALUATION LOOP STARTS
      DO 97 N=NSTART,NSTOP,2
      IF(LSTOP.EQ.1) GO TO 10
      WRITE(3,74) N
   74 FORMAT('1',/,'0',' NUMBER OF TAP POINTS=',I3,/,'0',' TAP GAIN RESO
     1LUTION= .0001')
      REFTD=0.5*(N-1)*T
      WRITE(3,66) REFTD
   66 FORMAT('0',' EQUALIZER REFERENCE TIME DELAY=',F9.6,' SECONDS')
      ICT=0
      JIT=0
```

```
      KZ=0
      KCT=0
      V=0.5*(N+1)
      IV=V+.1
      IVM=IV-1
      IVP=IV+1
      RES=RESOR
      PRE=ERINIT
      KFIR=0
      KN=0
      KSOL=0
      P=R*1.2
      NCO=0
      IF(NOP.NE.1) GO TO 1103
      KSOL=1
      P=R
      IF(JSTOP.EQ.0.OR.KSTOP.EQ.0) GO TO 1103
C         IN THE OPERATIONAL MODE, IF BOTH AMPLITUDE AND DELAY SPECS ARE
C         MET FOR A CERTAIN NUMBER OF TAP POINTS (N), LSTOP IS SET =1
C         PERFORMANCE IS THEN CHECKED FOR (N+2) TAPS. THEN THE RUN IS
C         TERMINATED.
      LSTOP=1
 1103 CONTINUE
C         SLOPE SPECIFICATION ALGORITHM. THE WEIGHTED SQUARED
C         CHANNEL AMPLITUDE RESPONSE AT +/- F/8 HZ FROM EACH F HZ
C         INCREMENT FREQUENCY LOCATION IS CONSIDERED.
C         THE COEFFICIENT MATRIX, A( , ), FOR THE SYSTEM OF SIMULTANEOUS
C         LINEAR EQUATIONS IS COMPUTED.
      DO 5 NN=1,N
      AA(NN)=0.0
      DO 1 I=MSF,LIM,MSF
    1 AA(NN)=AA(NN)+W(I)*(((.75*X(I)+.25*X(I-LAM))**2)*COS((NN-1)*(S*I-D
     1S))+((.75*X(I)+.25*X(I+LAM))**2)*COS((NN-1)*(S*I+DS)))
    5 CONTINUE
      WRITE(3,280)
  280 FORMAT('0',' MATRIX COEFFICIENTS')
      WRITE(3,281) (AA(I),I=1,N)
```

```
  281 FORMAT(' ',4(/,' ',10E13.4))
  604 LCT=0
      DO 9 J=1,N
      DO 6 K=1,J
      MCT=J-K+1
      LCT=LCT+1
    6 A(LCT)=AA(MCT)
    9 CONTINUE
C          THE WEIGHTED DIFFERENCE BETWEEN THE CHANNEL PHASE AND THE
C          IDEAL CHANNEL PHASE AT +/- F/8 HZ FROM EACH FREQUENCY LOCATION
C          IS CONSIDERED.
C          THE CONSTANT VECTOR, B( ), FOR THE SYSTEM OF EQNS. IS COMPUTED
      DO 8 MK=1,N
      B(MK)=0.0
      DO 7 J=MSF,LIM,MSF
    7 B(MK)=B(MK)+W(J)*((.75*X(J)+.25*X(J-LAM))*COS(P*(J-Q)-(TH(J)-D(J)*
     2DW)-(MK-V)*(S*J-DS))+(.75*X(J)+.25*X(J+LAM))*COS(P*(J+Q)-(TH(J)+D(
     3J)*DW)-(MK-V)*(S*J+DS)))
    8 CONTINUE
      IF(KSOL.EQ.0) GO TO 605
      WRITE(3,282)
  282 FORMAT('0',' CONSTANT VECTOR')
      WRITE(3,281) (B(I),I=1,N)
  605 CALL GELS(B,A,N,NCOL,EPS,IER,AUX)
C          THE TAP GAINS SOLUTION VECTOR, B( ), FOR THE EQUALIZER IS
C          OBTAINED FROM THE SYSTEM OF EQNS. USING SUBROUTINE GELS.
      WRITE(3,25) IER
   25 FORMAT('0',' IER=',I4)
      IF(IER.EQ.0) GO TO 15
C          IF NO SOLUTION IS OBTAINED A SMALLER VALUE OF THE IDEAL CHAN-
C          NEL DELAY (CALLED P THROUGHOUT MAIN LOOP) IS TRIED.
      P=P/2.
      NCO=NCO+1
      IF(NCO.LE.4) GO TO 604
      IF(IER) 98,15,99
C          THE LARGEST MODULUS TAP GAIN IS IDENTIFIED AS BNMAX
C          THE SUMMING AMPLIFIER GAIN, G, IS DEFINED.
```

```
   15 BNMAX=ABS(B(1))
      DO 16 JJ=2,N
      IF(ABS(B(JJ)).LE.BNMAX) GO TO 16
      BNMAX=ABS(B(JJ))
   16 CONTINUE
      G=BNMAX*.99999
      IF(KSOL.NE.1) GO TO 19
      WRITE(3,13)
   13 FORMAT('0',' TAP GAINS SOLUTION')
      WRITE(3,12) (B(I),I=1,N)
   12 FORMAT(' ',5(/,' ',7E17.7))
C          TAP GAIN MOMENTS ARE COMPUTED.
      TMOM1=0.0
      TMOM2=0.0
      TMOM3=0.0
      DO 1009 MK=1,N
      DIST=MK-IV
      TMOM1=TMOM1+ABS(B(MK)*DIST)
      TMOM2=TMOM2+(B(MK)**2)*ABS(DIST)
      TMOM3=TMOM3+(DIST**2)*ABS(B(MK))
 1009 CONTINUE
      WRITE(3,1010) TMOM1,TMOM2,TMOM3
 1010 FORMAT('0',' TAP GAIN MOMENTS.   FIRST=',E11.5,'   SECOND=',E11.5,
     1'   THIRD=',E11.5)
   19 CONTINUE
C          THE TAP GAINS ARE NORMALIZED USING THE LARGEST MODULUS, BNMAX.
C          TAP GAINS AND SUMMING AMPLIFIER GAINS ARE QUANTIZED USING A
C          RESOLUTION OF .0001 . TAP GAIN FACTORS THEN RANGE BETWEEN
C          -1. AND +1. IN VALUE.
      DO 17 LN=1,N
      B(LN)=B(LN)/G
      INB=B(LN)*10000.
   17 B(LN)=INB*0.0001
      ING=G*10000.
      G=ING*0.0001
      IF(KSOL.NE.1) GO TO 40
   18 WRITE(3,76)
```

```
   76 FORMAT('0',' TAP GAIN FACTORS')
      WRITE(3,77) (B(LF),LF=1,N)
   77 FORMAT(' ',4(/,' ',10F12.5))
      WRITE(3,78) G
   78 FORMAT('0',' SUMMING AMPLIFIER GAIN=',F10.5)
C          CHECK FOR UNREQUIRED TAP POINTS.
      ITPM=IVM
      DO 3000 ITP=1,IVM
      IF(B(ITP).EQ.0.0.AND.B(N+1-ITP).EQ.0.0) GO TO 3000
      ITPM=ITP-1
      GO TO 3001
 3000 CONTINUE
 3001 WRITE(3,3002) ITPM
 3002 FORMAT('0',' THE FOLLOWING NUMBER OF TAP POINTS ON EACH END OF THE
     1 EQUALIZER HAVE 0.0 GAIN FOR THIS RESOLUTION AND ARE NOT REQUIRED'
     2,I3)
   40 CONTINUE
C          Y(L) IS PROPORTIONAL TO THE DELAY AT EQUALIZER TAP L RELATIVE
C          TO THE EQUALIZER MIDPOINT, TAP IV. PERFECT DELAY OF T SECONDS
C          PER SECTION IS ASSUMED HERE.
      DO 14 L=1,N
   14 Y(L)=(L-IV)*3.141593*F*T/LAM
      IF(F.LT.200.) GO TO 42
   41 FINC=8.
      NINC=8*M
      GO TO 44
   42 FINC=4.
      NINC=4*M
   44 CONTINUE
      ER=0.0
      NB=0
      SUMBN=0.0
      DO 45 KK=1,N
      SUMBN=SUMBN+B(KK)
   45 CONTINUE
      IF(KSOL.NE.1.OR.KCT.EQ.1) GO TO 43
      ZERES=G*SUMBN
```

```
C          THE EQUALIZER AMPLITUDE RESPONSE AT ZERO HZ FREQUENCY IS ZERES
C          IF IT IS NEGATIVE, DC POLARITY REVERSAL IS APPARENT.
      WRITE(3,1097) ZERES
 1097 FORMAT('0',' ZERO HZ EQUALIZER AMPLITUDE RESPONSE=',F10.4)
   43 CONTINUE
      IF(SUMBN.GE.0.) GO TO 46
      NB=-1
   46 CONTINUE
C          THE FIRST FREQUENCY SUB-INCREMENT IS DIVIDED INTO 8 EQUAL FINE
C          INCREMENTS. FOR THE FIRST OF THESE, THE REAL, CR(1), AND THE
C          IMAGINARY,CI(1), PARTS OF THE EQUALIZER CHARACTERISTIC ARE
C          COMPUTED. THE PHASE ANGLE IS CLOSE TO THAT AT DC.
      CR(1)=0.0
      CI(1)=0.0
      DO 202 NS=1,N
      CR(1)=CR(1)+B(NS)*G*COS(Y(NS)/16.)
      CI(1)=CI(1)+B(NS)*G*SIN(Y(NS)/16.)
  202 CONTINUE
      SANG=ATAN(CI(1)/CR(1))+A2*NB
      SMAG=SQRT(CR(1)**2+CI(1)**2)
      IF(KSOL.NE.1) GO TO 601
      IF(KCT.NE.0) GO TO 601
      WRITE(3,79)
   79 FORMAT('0',5X,'EQUALIZER',/,'0',3X,'I',3X,'NB',4X,'REAL RESPONSE',
     13X,'IMAG. RESPONSE',5X,'PHASE ANGLE',7X,'MAGNITUDE')
      WRITE(3,80) NCOL,NB,CR(1),CI(1),SANG,SMAG
   80 FORMAT(' ',I4,I5,4E17.7)
  601 CONTINUE
C          THE REAL AND IMAGINARY PARTS OF THE EQUALIZER CHARACTERISTIC
C          ARE COMPUTED AT EACH FINE INCREMENT. THE PHASE ANGLE CHANGE IS
C          ASSUMED TO BE LESS THAN 90 DEGREES. IN THIS WAY THE BRANCH
C          AMBIGUITY OF THE ATAN IS REMOVED. THE EQUALIZER CHARACTERISTIC
C          MAGNITUDE, SMAG, AND ANGLE, SANG, ARE COMPUTED.
      DO 276 I=2,8
      CR(I)=0.0
      CI(I)=0.0
      DO 204 IL=1,N
```

```
      CR(I)=CR(I)+B(IL)*G*COS(I*Y(IL)/16.)
      CI(I)=CI(I)+B(IL)*G*SIN(I*Y(IL)/16.)
  204 CONTINUE
      IF(CR(I-1)) 206,205,206
  205 IF(CI(I)) 207,209,208
  207 IF(CR(I)) 260,270,251
  208 IF(CR(I)) 251,270,260
  206 IF(CI(I)) 210,209,210
  209 IF(CR(I)) 260,249,260
  210 IF(CR(I)/CR(I-1)) 212,213,260
  212 IF(CI(I)) 214,209,215
  214 IF(CR(I)-CR(I-1)) 250,260,251
  213 IF(CI(I)) 216,249,217
  216 IF(CR(I-1)) 270,270,269
  217 IF(CR(I-1)) 269,270,270
  215 IF(CR(I)-CR(I-1)) 251,260,250
  249 SANG=0.0
      GO TO 275
  250 NB=NB-1
      GO TO 260
  251 NB=NB+1
  260 SANG=ATAN(CI(I)/CR(I))+NB*A2
      GO TO 275
  269 NB=NB-1
  270 SANG=A1+NB*A2
  275 SMAG=SQRT(CR(I)**2+CI(I)**2)
      IF(KSOL.NE.1) GO TO 602
      IF(KCT.NE.0) GO TO 602
      WRITE(3,80) I,NB,CR(I),CI(I),SANG,SMAG
  602 CONTINUE
  276 CONTINUE
C         THE EIGHTH FINE FREQUENCY INCREMENT CHARACTERISTIC DEFINES THE
C         FIRST SUB-INCREMENT CHARACTERISTIC.
      CR(1)=CR(8)
      CI(1)=CI(8)
      L4=1
      LTOG=2
```

```
      DO 176 I=2,NINC
      LTOG=LTOG+2
      L4=L4+1
      PRAN=SANG
C        COMPUTATION OF THE EQUALIZER CHARACTERISTIC REAL AND IMAGINARY
C        COMPONENTS AND THEN MAGNITUDE AND ANGLE PROCEEDS AT EACH SUB-
C        INCREMENT IN FREQUENCY, AS ABOVE.
      CR(I)=0.0
      CI(I)=0.0
      DO 104 IL=1,N
      CR(I)=CR(I)+B(IL)*G*COS(I*Y(IL)/2.)
      CI(I)=CI(I)+B(IL)*G*SIN(I*Y(IL)/2.)
  104 CONTINUE
      IF(CR(I-1)) 106,105,106
  105 IF(CI(I)) 107,109,108
  107 IF(CR(I)) 160,170,151
  108 IF(CR(I)) 151,170,160
  106 IF(CI(I)) 110,109,110
  109 IF(CR(I)) 160,149,160
  110 IF(CR(I)/CR(I-1)) 112,113,160
  112 IF(CI(I)) 114,109,115
  114 IF(CR(I)-CR(I-1)) 150,160,151
  113 IF(CI(I)) 116,149,117
  116 IF(CR(I-1)) 170,170,169
  117 IF(CR(I-1)) 169,170,170
  115 IF(CR(I)-CR(I-1)) 151,160,150
  149 SANG=0.0
      GO TO 175
  150 NB=NB-1
      GO TO 160
  151 NB=NB+1
  160 SANG=ATAN(CI(I)/CR(I))+NB*A2
      GO TO 175
  169 NB=NB-1
  170 SANG=A1+NB*A2
  175 SMAG=SQRT(CR(I)**2+CI(I)**2)
      IF(KSOL.NE.1) GO TO 603
```

```
      IF(KCT.NE.0) GO TO 603
      WRITE(3,80) I,NB,CR(I),CI(I),SANG,SMAG
  603 CONTINUE
      IF(LTOG.LT.3) GO TO 176
      LTOG=0
      J2=(I+.1)/2.
      IF(KSOL.EQ.0.OR.KCT.EQ.1) GO TO 614
C         THE EQUALIZER DIFFERENTIAL DELAY IS OBTAINED (FOR PRINTOUT
C         ONLY) BY A NUMERICAL DIFFERENTIATION OF ITS PHASE SHIFT FCN.
      E(J2)=(SANG-PRAN)*FINC/(6.283185*F)
      CM(J2)=SMAG
      CA(J2)=SANG
C         EQUALIZED CHANNEL PHASE SHIFT, HA( ), IS COMPUTED AT EACH EVEN
C         NUMBERED SUB-INCREMENT FREQUENCY.
  614 HA(J2)=TH(J2)+SANG
      IF(F.LT.200.) GO TO 177
      IF(L4.NE.4) GO TO 176
      L4=0
C         EQUALIZED CHANNEL AMPLITUDE RESPONSE, HM( ), AND THE EQUALIZED
C         CHANNEL WEIGHTED ERROR ACCUMULATION, ER, ARE COMPUTED AT EACH
C         F/2 HZ FREQUENCY LOCATION. P IN THE ERROR EQUATION IS RELATED
C         TO THE IDEAL CHANNEL PHASE SLOPE BEING CONSIDERED.
  177 HM(J2)=X(J2)*SMAG
      ER=ER+((HM(J2)*COS(HA(J2))-COS(P*J2))**2+(HM(J2)*SIN(HA(J2))-SIN(P
     4*J2))**2)*W(J2)*6.283185*F
  176 CONTINUE
      ERNR=SCNOR*SQRT(ER)
      ERNRPC=100.*ERNR
      IF(KFIR.NE.0) GO TO 618
      ER1=ER
      KFIR=1
  618 IF(KCT.EQ.1) GO TO 613
      PHPT=P*NSL
      WRITE(3,609) P,PHPT,ER,ERNR,ERNRPC
  609 FORMAT('0',' P=',F8.4,5X,'PHPT=',F8.4,5X,'ER=',E11.5,'  ERNR=',E11
     1.5,'   ERNRPC=',F7.2)
      IF(KSOL.EQ.1) GO TO 610
```

```
      IF(P.EQ.0.0) GO TO 616
C         IN THE INVESTIGATION MODE, IF A LOCAL MINIMUM OF EQUALIZED
C         CHANNEL ERROR VS. P HAS NOT BEEN LOCATED, THE VALUE OF P IS
C         FUTHER REDUCED. THE ENTIRE EQUALIZER TAP GAIN SOLUTION AND
C         ERROR EVALUATION IS REPEATED. THIS PROCESS IS OMITTED IN THE
C         OPERATIONAL MODE, WHERE P=R=SLOPE FOR ACCEPTABLE SUMPHE IS
C         THE ONLY VALUE USED.
      P=P/1.025
      JIT=JIT+1
C         NO MORE THAN 40 ITERATIONS ON P ARE PERMITTED.
      IF(JIT.GT.40) GO TO 619
      IF(ER-PRE) 606,606,607
  606 PRE=ER
      KN=KN+1
      GO TO 604
  615 PRE=ER
      GO TO 604
  607 IF(KN.LE.1) GO TO 615
  619 IF((PRE-ER1).LT.100.) GO TO 617
      P=R
      GO TO 616
C         WHEN A LOCAL ERROR MINIMUM IS APPARENT P IS RESET TO THE PRE-
C         VIOUS VALUE OR TO THE VALUE OF R, ABOVE.
  617 P=(P*1.025)*1.025
C         A SOLUTION FOR P HAS BEEN FOUND. SETTING KSOL=1 INDICATES THIS
  616 KSOL=1
C         THE NON-EQUALIZED CHANNEL ERROR CORRESPONDING TO THE CHOSEN
C         VALUE OF P IS COMPUTED.
      ERIN=0.0
      DO 608 K=LAM,LIM,LAM
  608 ERIN=ERIN+((X(K)*COS(TH(K))-COS(P*K))**2+(X(K)*SIN(TH(K))-SIN(P*K)
     1)**2)*W(K)*6.283185*F
      EIPNOR=SCNOR*SQRT(ERIN)
      EIPNPC=100.*EIPNOR
      WRITE(3,73) ERIN,EIPNOR,EIPNPC
   73 FORMAT('0',' CHANNEL P-SLOPE RAW SQUARED ERROR=',E11.5,'   SCALED,
     1 NORMALIZED ERROR=',E11.5,'   IN PERCENT=',F7.2)
```

```
C          WITH KSOL=1, THE EQUALIZATION IS REPEATED FOR THE CHOSEN VALUE
C          OF P, AND PRINTOUT OF THE EQUALIZER CHARACTERISTIC AT EVERY
C          FREQUENCY CONSIDERED IS OBTAINED.
      GO TO 604
C          THE IDEAL CHANNEL ABSOLUTE ENVELOPE DELAY, DIABS, IS COMPUTED
  610 DIDABS=P*MSF/(6.283185*F)
      WRITE(3,611) DIDABS
  611 FORMAT('0',' IDEAL CHANNEL DIFFERENTIAL ENV. DELAY ZERO=',F9.6,' S
     1ECONDS')
      WRITE(3,81)
   81 FORMAT('0',' EQUALIZER AMPLITUDE RESPONSE')
      WRITE(3,82) (CM(I),I=1,LIM)
   82 FORMAT(' ',7(/,' ',10E13.4))
      WRITE(3,83)
   83 FORMAT('0',' EQUALIZER PHASE RESPONSE')
      WRITE(3,82) (CA(J),J=1,LIM)
      WRITE(3,84)
   84 FORMAT('0',' EQUALIZER ABSOLUTE ENVELOPE DELAY, SECONDS')
      WRITE(3,82) (E(MD),MD=1,LIM)
  613 WRITE(3,85)
   85 FORMAT('0',/,'0',' EQUALIZED CHANNEL AMPLITUDE RESPONSE')
      WRITE(3,82) (HM(K),K=LAM,LIM,LAM)
      IREF=(501./F)*MSF
      IREF=IREF*2
      REF=HM(IREF)
      RGAIN=20.*ALOG10(REF)
      WRITE(3,86) RGAIN
   86 FORMAT('0',' EQUALIZED CHANNEL GAIN, DB, 1000 HZ,=',F9.3)
      DO 95 LR=LAM,LIM,LAM
   95 HM(LR)=20.*ALOG10(HM(LR)/REF)
      WRITE(3,87)
   87 FORMAT('0',' EQUALIZED CHANNEL AMPLITUDE RESPONSE, DB, 1000 HZ REF
     1.')
      WRITE(3,88) (HM(LDB),LDB=LAM,LIM,LAM)
   88 FORMAT(' ',7(/,' ',10F13.4))
      WRITE(3,89)
   89 FORMAT('0',' EQUALIZED CHANNEL PHASE RESPONSE, RADIANS')
```

```
      WRITE(3,88) (HA(L),L=1,LIM)
C          THE EQUALIZED CHANNEL ENVELOPE DELAY IS COMPUTED BY NUMERICAL
C          DIFFERENTIATION OF ITS PHASE SHIFT FUNCTION VS. FREQUENCY.
      DO 96 K=2,LUM
   96 E(K)=(HA(K+1)-HA(K-1))*LAM/(6.283185*F)
C          IN THE FREQUENCY RANGE 1000 TO 2600 HZ THE MINIMUM AND MAXIMUM
C          DELAYS ARE IDENTIFIED AS DLOW AND DHI. THE AVERAGE OF THE TWO,
C          DMID, IS DEFINED AS THE DELAY REFERENCE OR ZERO.
      DLOW=E(NL)
      DO 92 J=NL,NH
      IF(E(J).GE.DLOW) GO TO 92
      DLOW=E(J)
   92 CONTINUE
      DHI=E(NL)
      DO 93 J=NL,NH
      IF(E(J).LE.DHI) GO TO 93
      DHI=E(J)
   93 CONTINUE
      DMID=(DLOW+DHI)/2.
C          EQUALIZED CHANNEL DIFFERENTIAL DELAY RELATIVE TO DMID IS
C          COMPUTED (AT EVEN-NUMBERED SUB-INCREMENT FREQUENCIES EXCEPT
C          THE LOWEST AND THE HIGHEST)
      DO 91 L=2,LUM
   91 E(L)=E(L)-DMID
      WRITE(3,180)
  180 FORMAT('0',' EQUALIZED CHANNEL DIFFERENTIAL ENVELOPE DELAY CHAR.,
     1SECONDS')
      WRITE(3,181) DMID
  181 FORMAT(' ',' ZERO =',E12.4,' SECONDS')
      WRITE(3,82) (E(LK),LK=2,LUM)
      WRITE(3,182) ER,ERNR,ERNRPC
  182 FORMAT('0',' EQUALIZED CHANNEL RESIDUAL RAW SQUARED ERROR=',E11.5,
     1'   SCALED, NORMALIZED ERROR=',E11.5,'   IN PERCENT=',F7.2)
C          EQUALIZED CHANNEL AMPLITUDE RESPONSE IS CHECKED AGAINST C4
C          SPECS.
      KFA=0
      WRITE(3,183)
```

```
 183 FORMAT('0',' EQUALIZED CHANNEL AMPLITUDE RESPONSE FAILS TO MEET C4
    1 SPECS. AT THE FOLLOWING FREQUENCIES, HZ')
     DO 450 I=LAM,LIM,LAM
     FRE=(I*F)/MSF
     IF(FRE.LT.300.) GO TO 450
     IF(FRE.GE.500.) GO TO 451
     IF(HM(I).GE.0.) GO TO 452
 454 IF((HM(I)+6.).GE.0.) GO TO 450
 456 KFA=1
     WRITE(3,184) FRE
 184 FORMAT(' ','   LOSS,',F12.3)
     GO TO 450
 452 IF((HM(I)-2.).LE.0.) GO TO 450
     KFA=1
     WRITE(3,185) FRE
 185 FORMAT(' ','   GAIN,',F12.3)
     GO TO 450
 451 IF(FRE.GT.3000.) GO TO 453
     IF(HM(I).GE.0.) GO TO 452
     IF((HM(I)+3.).GE.0.) GO TO 450
     GO TO 456
 453 IF(FRE.GT.3200.) GO TO 450
     IF(HM(I).GE.0.) GO TO 452
     GO TO 454
 450 CONTINUE
     IF(KFA.EQ.1) GO TO 455
     WRITE(3,186)
 186 FORMAT(' ','   NONE')
     IF(KCT.EQ.0.OR.KZ.NE.0) GO TO 455
     JSTOP=1
 455 KFB=0
C         EQUALIZED CHANNEL DIFFERENTIAL ENVELOPE DELAY IS CHECKED
C         AGAINST SYMMETRICAL (ABOUT ZERO DELAY) C4 SPECS.
     WRITE(3,187)
 187 FORMAT('0',' EQUALIZED CHANNEL DIFFERENTIAL ENV. DELAY FAILS TO ME
    1ET SYMMETRICAL C4 SPECS. AT THE FOLLOWING FREQUENCIES, HZ')
     DO 460 I=2,LUM
```

```
      FRE=(I*F)/MSF
      IF(FRE.LT.500.) GO TO 460
      IF(FRE.GE.600.) GO TO 461
  467 IF(ABS(E(I)).LE.0.0015) GO TO 460
  463 IF(E(I)) 470,460,471
  461 IF(FRE.GE.800.) GO TO 462
      IF(ABS(E(I)).LE.0.00075) GO TO 460
      GO TO 463
  462 IF(FRE.GE.1000.) GO TO 464
  466 IF(ABS(E(I)).LE.0.00025) GO TO 460
      GO TO 463
  464 IF(FRE.GT.2600.) GO TO 465
      IF(ABS(E(I)).LE.0.00015) GO TO 460
      GO TO 463
  465 IF(FRE.LE.2800.) GO TO 466
      IF(FRE.LE.3000.) GO TO 467
      GO TO 460
  470 WRITE(3,188) FRE
  188 FORMAT(' ','  MINUS,',F12.3)
      GO TO 472
  471 WRITE(3,189) FRE
  189 FORMAT(' ','   PLUS,',F12.3)
  472 KFB=1
  460 CONTINUE
      IF(KFB.EQ.1) GO TO 473
      WRITE(3,186)
      IF(KCT.EQ.0.OR.KZ.NE.0) GO TO 473
      KSTOP=1
  473 CONTINUE
C          IF BOTH AMPLITUDE AND DELAY SPECS HAVE BEEN MET USING
C          RESOLUTION=RES (FOR PERFECT NOMINAL DEALY PER SECTION), JSTOP
C          AND KSTOP ARE RETAINED AS =1. IN THE OPERATIONAL MODE, ONE
C          MORE VALUE OF THE NUMBER OF TAP POINTS WILL BE EVALUATED
C          BEFORE THE CURRENT RUN IS AUTOMATICALLY TERMINATED.
      IF(JSTOP.EQ.1.AND.KSTOP.EQ.1) GO TO 474
      JSTOP=0
      KSTOP=0
```

```
  474 CONTINUE
      IF(KCT.EQ.0) GO TO 407
C         FOR THE SPECIFIED NUMBER OF SUB-RUNS, THE DELAY PER SECTION OF
C         THE EQUALIZER IS CHOSEN FROM THE GAUSSIAN DISTRIBUTION
C         USING SUBROUTINE GAUSS, WHICH EMPLOYS A RANDOM NUMBER GENERAT-
C         OR. SUMZ IS PROPORTIONAL TO THE DELAY AT THE TAP POINT WITH
C         RESPECT TO THE MID POINT. Y( ) IS THE MATHEMATICAL EXPRESSION
C         FOR THIS DELAY AS USED IN THE EQUALIZER EVALUATION.
      IF(KZ.GE.KVAR) GO TO 502
      KZ=KZ+1
      WRITE(3,190) KZ,N,AM,SD,REJ
  190 FORMAT('1',' DELAY SECTION VARIATION SUB-RUN NO.',I3,', FOR',I3,'
     1TOTAL TAP POINTS.   MEAN=',F5.2,'  STD. DEV.=',F5.2,'  REJ=',F5.2,
     2/,'0',' VALUES OF Z AND SUMZ',/,'0',' NEGATIVE SIDE TAPS')
      SUMZ=0.0
      Y(IV)=0.0
      DO 500 K=1,IVM
  503 CALL GAUSS(IXE,SD,AM,Z)
      IF(ABS(Z-AM).GT.REJ) GO TO 503
      SUMZ=SUMZ+Z
      WRITE(3,191) Z,SUMZ
  191 FORMAT(' ',2F10.4)
  500 Y(IV-K)=Y(IV-K+1)-(3.141593*F*T/LAM)*Z
      WRITE(3,192)
  192 FORMAT('0',' POSITIVE SIDE TAPS')
      SUMZ=0.0
      DO 501 K=IVP,N
  504 CALL GAUSS(IXE,SD,AM,Z)
      IF(ABS(Z-AM).GT.REJ) GO TO 504
      SUMZ=SUMZ+Z
      WRITE(3,191) Z,SUMZ
  501 Y(K)=Y(K-1)+(3.141593*F*T/LAM)*Z
      GO TO 44
C         IN THE INVESTIGATION MODE, IF LOP=0 THE PERFORMANCE IS DETER-
C         MINED FOR RESOLUTION=RES*10 AND THEN RES*40, WHERE RES IS THE
C         ORIGINALLY SPECIFIED RESOLUTION.
  502 IF(LOP.EQ.1) GO TO 97
```

```
      IF(ICT-1) 404,408,97
  404 ICT=1
      RES=RES*10.
      GO TO 409
  407 KCT=1
      IF(KOP.EQ.1) GO TO 97
      GO TO 409
  408 ICT=2
      RES=RES*4.
  409 WRITE(3,193) RES,N
  193 FORMAT('1',' TAP GAIN RESOLUTION=',F7.5,5X,'TOTAL NUMBER OF TAP PO
     1INTS=',I2)
      DO 406 I=1,N
      INTB=B(I)/RES
  406 B(I)=INTB*RES
      INTG=G/RES+.5
      G=INTG*RES
      GO TO 18
   10 NEQUL=N-4
      WRITE(3,1007) NEQUL
 1007 FORMAT('1',' EQUALIZATION REQUIREMENTS WERE MET USING ',I3,' TAP P
     1OINTS.  RUN IS COMPLETE')
      GO TO 100
   98 WRITE(3,194)
C          EXIT MESSAGE AFTER FIVE FAILURES TO SOLVE SIMULTANEOUS EQNS
C          USING GELS. INDICATES PIVOT ELEMENT IN ELIMINATION IS ZERO.
  194 FORMAT('0',' REALLY AMISS')
      GO TO 97
   99 WRITE(3,195)
C          EXIT MESSAGE AFTER FIVE FAILURES TO SOLVE SIMULTANEOUS EQNS
C          USING GELS. INDICATES POSSIBLE LOSS OF SIGNIFICANCE ON STEP
C          IER+1 (IER IS PRINTED).
  195 FORMAT('0',' SINGULAR SET OF EQUATIONS')
   97 CONTINUE
  100 CONTINUE
      STOP
      END
```